



**Clockhouse Primary School**  
**Maths Calculation Policy**



**We believed we could, so we did!**

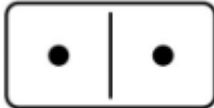



This policy aims to demonstrate the approach to teaching mathematical concepts and strategies to ensure a consistent approach across the school. The five main areas included are the four operations (addition, subtraction, multiplication and division) and fractions. Although the strategies are organised into separate year groups, children will be working at their own pace and learning should be adapted to meet the needs of the individual, this could mean teaching strategies from the year group below in order to build a secure foundation to build upon. All children should be given the tools and support they need in order to be successful. When teaching each concept, close attention should be paid to the language used to explain the process and children should be encouraged to use the key vocabulary when talking about their own learning.

Concrete Pictorial Abstract- CPA

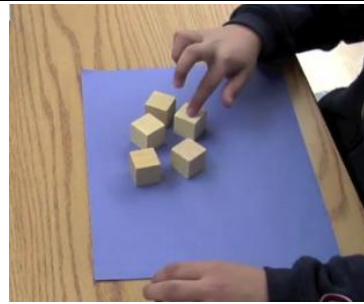


**All children** should be given the opportunity to explore mathematical concepts through the use of manipulatives to develop and deepen their understanding. The link between the concrete resource and the pictorial representation should be made explicit to the children in order to support the transition from concrete resources to pictorial markings. The children should be shown the symbols which represent the maths within the markings to support the transition to the most efficient approach.

EYFS

| Objective   | Resources and representations  |
|---|--|
| <b>3- and 4-Year olds will be taught to ...</b>   |  |
| Develop fast recognition of up to 3 objects, without having to count them individually ('subitising') |     |

Know that the last number reached when counting a small set of objects tells you how many there are in total ('cardinal principle')

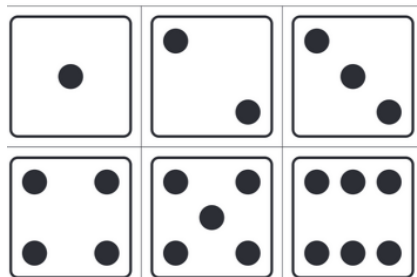


Link numerals and amounts: for example, showing the right number of objects to match the numeral, up to 5

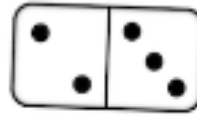
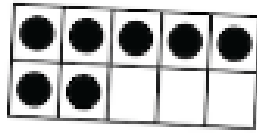


### Children in Reception will be taught to ...

Subitise



- Using dot cards, dominoes and dice as part of a game, including irregularly arranged dots (e.g. stuck on)
- Playing hidden objects games where objects are revealed for a few seconds, for example, small toys hidden under a bowl – shuffle them, lift the bowl briefly and ask how many there were
- 'All at once fingers' – show me four fingers.

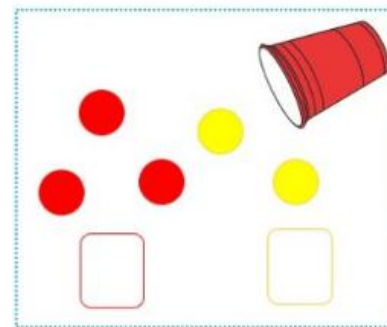
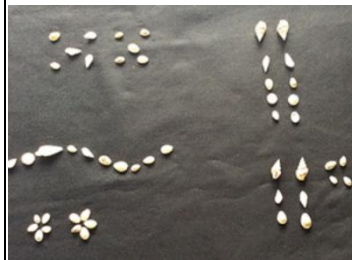


Link the number symbol (numeral) with its cardinal number value

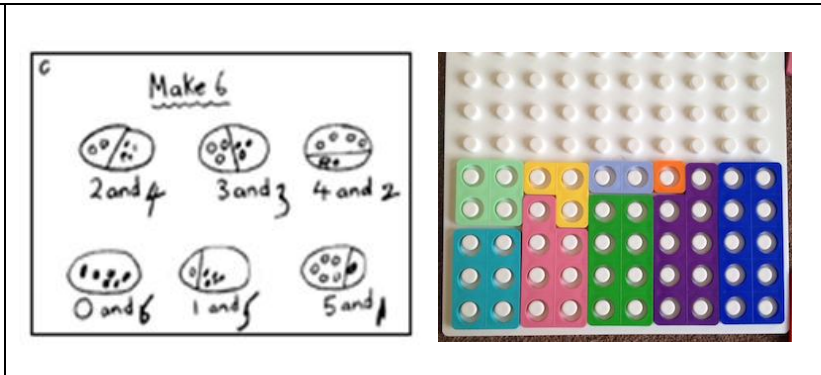
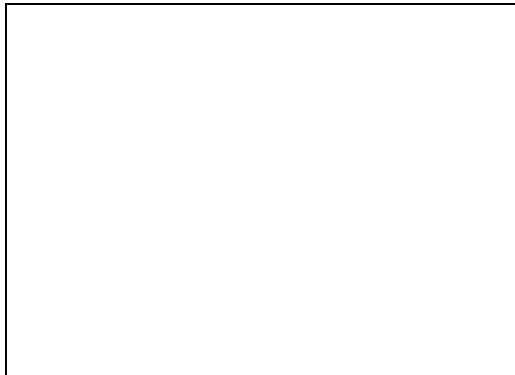


- Counting things of different sizes – this helps children to focus on the numerosity of the count
- Counting things that can't be seen, such as sounds, actions, words
- Counting things that cannot be moved, such as pictures on a screen, birds at the bird table, faces on a shape.
- Playing dice games to collect a number of things
- Playing track games and counting along the track.
- 

Explore the composition of numbers to 10.

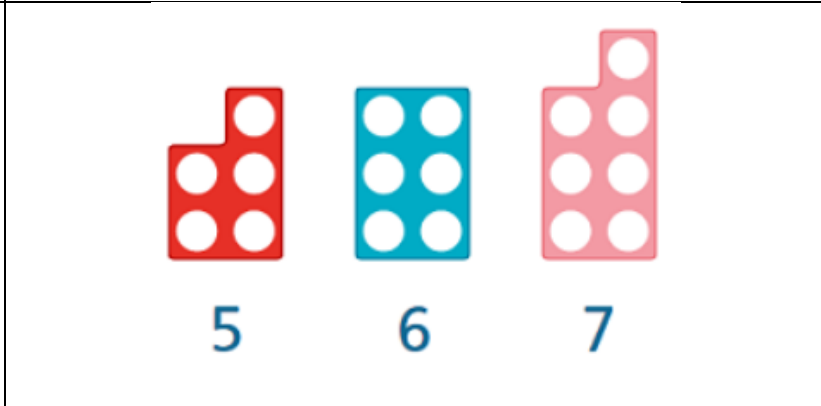


- Encouraging making arrangements with (e.g.) ten. Ensure the children talk about the different arrangements they can see within the whole
- Numicon towers: layering up Numicon pieces of the same total
- Putting things into two containers in different ways
- Making a number with two different kinds of things. For example, make a fruit skewer with five pieces of fruit, using bowls of bananas/strawberries to choose from; then ask the children to describe how they have made theirs. They should compare it with a partner's: 'What is the same about your skewers? What is different?'



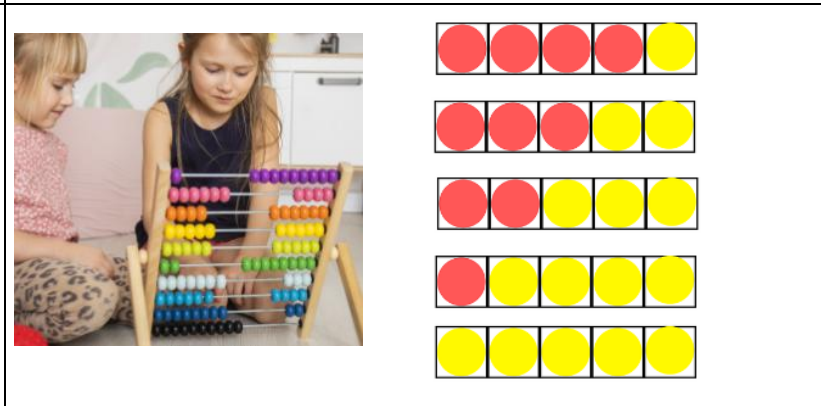
- Bunny Ears: using your fingers like bunny ears. 'With two hands, show me five fingers. Can you do it in a different way?' Or, 'Show five fingers altogether with a friend'
- Spill the Beans: using double-sided counters or beans, where one side is coloured, throw the collection and note how many of each type can be seen and how many altogether
- using six bean bags with different fabric on each side, throw the collection and note how many of each type can be seen.

Understand the 'one more than/one less than'

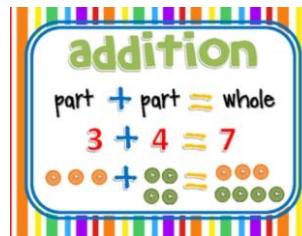
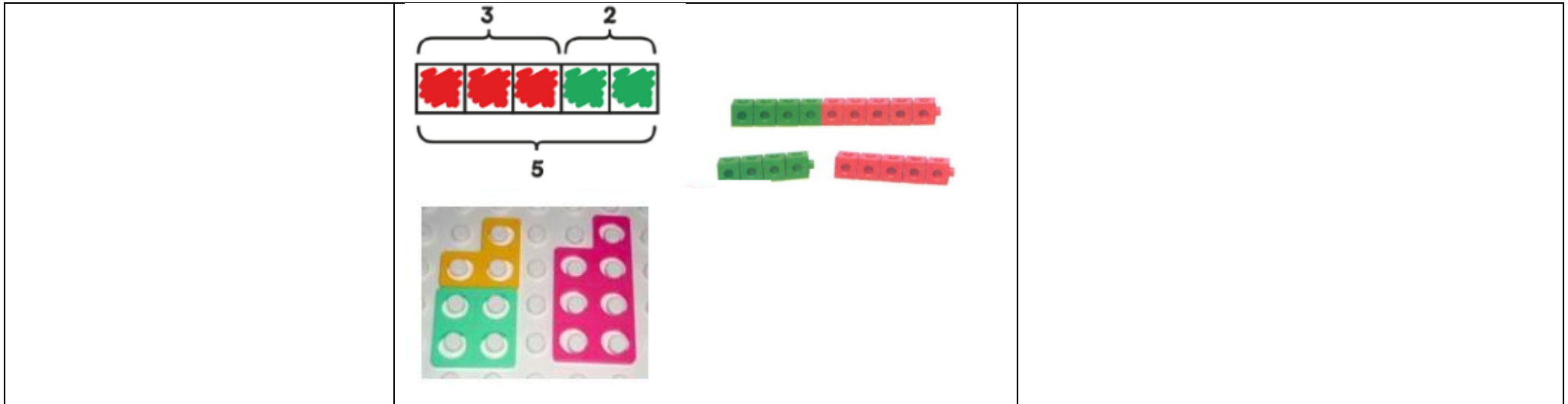


- Labelling groups with the correct numeral. Do children spot the error if a group is mislabelled? For example, 'The label on the pot says 4 and we have 5 – what do we need to do?' A child may say, 'We need to take one out because we have one too many.'
- Ensuring children focus on the numerosity of the group by having items in the collection of different kinds and sizes
- Making predictions about what the outcome will be in stories, rhymes and songs if one is added to, or if one is taken away.

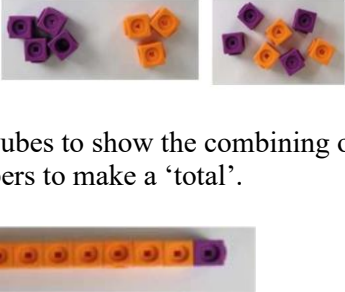
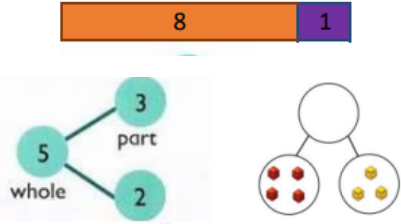
Automatically recall number bonds for numbers 0–5 and some to 10.

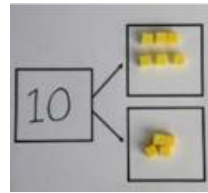


- Playing hiding games with a number of objects in a box, under a cloth, in a tent, in a cave, etc.
- Utilising classroom routines such as tidy-up time to identify how many are still missing from a pot with a number label.

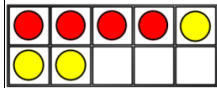


Year 1

| Objective & Strategy  | Concrete   | Pictorial   | Abstract   |
|---|--|---|--|
| <p>Combining two parts to make a whole<br/>Adding two 1-digit numbers</p> |  <p>Use cubes to show the combining of two numbers to make a 'total'.</p> <p>Using two separate colours to show the children each part<br/>Orange (part) and purple (part) = whole</p> |  | <p><math>4 + 3 = 7</math>    <math>7 = 3 + 4</math><br/>4 is a part, 3 is a part, 7 is the whole</p> <p>Include missing number calculations to deepen the children's understanding</p> <p><math>7 = ? + 4</math></p> <p><math>3 + ? = 7</math></p> |



Encourage the children to organise the combining of two numbers to see the two separate parts

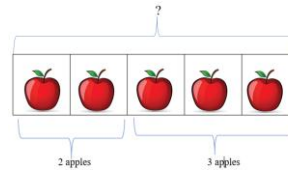


Using a tens frame can help to develop the children's understanding of regrouping to make 10.

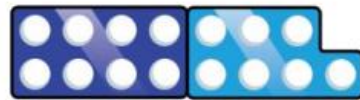


$$1 + 3 = 4$$

Children should be taught to use their own markings to demonstrate the parts within the calculation.



Adding 1-digit and 2-digit numbers to 20  
Starting at the bigger number and counting on

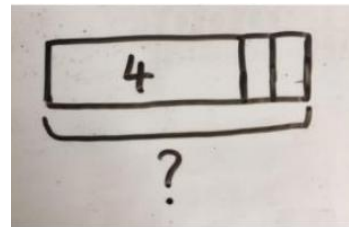


Children should be encouraged to find the total using concrete objects to begin with, such as cubes. To move on to a more efficient method, they can then use Numicon shapes to combine two numbers.

Children should be taught to start with the biggest number and count on (more efficient as less counting on/jumps to make)



The children can use cubes or Numicon to support their understanding.



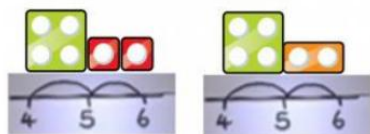
In order to bridge the gap between counting both sets of objects and 'counting on', show the children a bar model with the largest number already inside. They then need to 'count on'.



Encourage the children to circle the largest number in the calculation and on the number line.

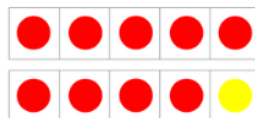
Children should be taught to place the larger number in their head and count on to find the total. They should be using numbers and symbols to represent the calculation.

Provide opportunities to test the children by starting the calculation with the smallest number-will they remember to start with the largest number?

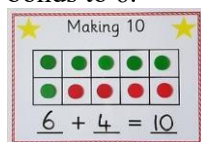


Showing the children the numbers represented by the concrete resources will allow them to make the link. This can be replicated using a bar model, with the total written in the bar and additional cubes added to find the total so the children are just counting on the number they are adding.

Represent and use number bonds and related subtraction facts within 20



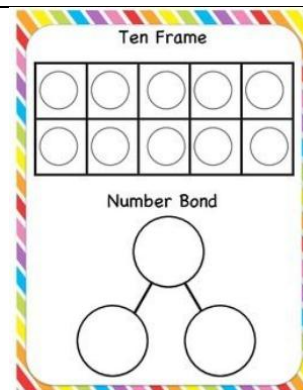
Provide opportunities for the children to explore the concept of number bonds. Using ten frames to explore number bonds to 10, egg boxes for number bonds to 6.



Children should be given the opportunity to use the Numicon pieces to find number bonds to numbers up to and including 20



Numberblocks can be used in conjunction with Mastering Number



The children can move on to using colours to represent the bonds.



The children should then be able to write their number bonds using numbers and symbols-

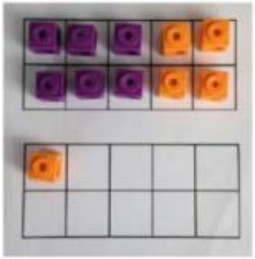
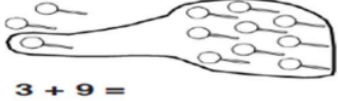
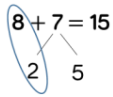
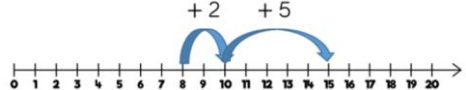
$$6 + 4 = 10$$

$$4 + 6 = 10$$

$$10 - 4 = 6$$

$$10 - 6 = 4$$

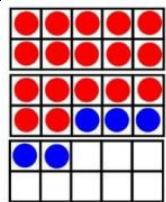
They should begin to understand the relationship between the numbers, including the commutative relationship of addition. If they know that 6 needs 4 to make 10, then 4 will need 6 to make 10.

|  |   |   |   |
|--|---|---|---|
|  |   |   |   |
| <p>Regrouping to make 10<br/><i>Essential for column addition as they progress</i></p>   |  <p>When adding 6 and 5, the children should be encouraged to 'make 10'. To develop this and allow the children opportunities to 'pattern spot' the children should be given the time to explore using tens frames. If they fill the tens frame using the 5, they will make 10 and have 1 left over.</p>   | <p>The children can draw their own tens frame to support their understanding.</p>  <p>If the children use markings, they will be able to group 10. Recognising that 9 needs 1 more to make 10. Then there will be 2 left. This is easier to calculate <math>10 + 2</math>, rather than <math>3 + 9</math>.</p> | <p>This is a crucial skill for the children to learn as they begin their journey in mathematics as it will help them to become more efficient and effective later on with more complex calculations and problems.</p>  <p>In this example, the children should begin to see the 'bond to 10'. Now they need to add 10 and 5.</p>  |
| <p>Vocabulary</p>  | <p><b>Stem Sentences</b></p>  |   |   |
| <p>addition, add, more, plus, and, make sum, total, altogether one more, two more, ten more equals, equal to, count on part, part, whole is the same as number bonds/pairs</p> | <p><i>The ___ represents all of the counters.</i><br/> <i>The ___ represents the ___ counters.</i><br/> <i>The whole is ___ and one part is ___ so the other part must be ___.</i><br/> <i>There are ___ and ___.</i><br/> <i>We can write this as ___ plus ___.</i><br/> <i>The ___ represents the ___.</i><br/> <i>___ is equal to ___ plus ___.</i><br/> <i>___ plus ___ is equal to ___.</i><br/> <i>If we change the order of the addends, the sum remains the same'</i></p> |   |   |

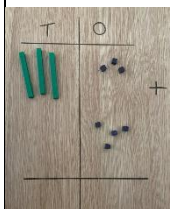
Year 2

|                      |          |           |          |
|----------------------|----------|-----------|----------|
| Objective & Strategy | Concrete | Pictorial | Abstract |
|----------------------|----------|-----------|----------|

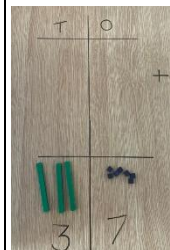
Add a two-digit number and ones



$17 + 5 = 22$   
Use a ten frame to highlight the move into the next ten.

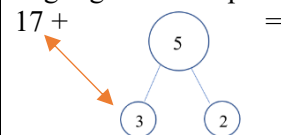


Represent the numbers using dienes.

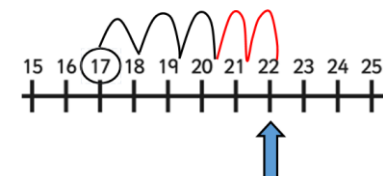
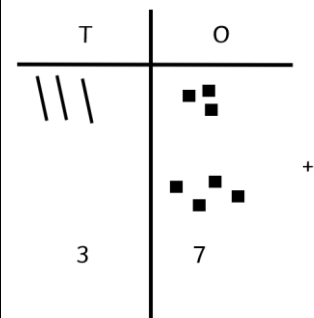


Start with the smallest digits in values (ones) count the total e.g.  $3 + 4 = 7$   
Children to recognise that they are 'combing' the two numbers to find the total.

Highlight the composition of number



$7 + 3 = 10$



The children do not need to use different colours, this is to highlight the composition of 5. As it is made of 3 and 2. If we add 3, this is a number bond to 10. Then we have 2 more left.

|   |   |   |
|---|---|---|
|   | T | O |
|   | 3 | 3 |
| + |   | 4 |
|   | 3 | 7 |

Add a two-digit number and tens



$25 + 10 = 35$   
Encourage the children to notice (or highlight) that the ones do not change.

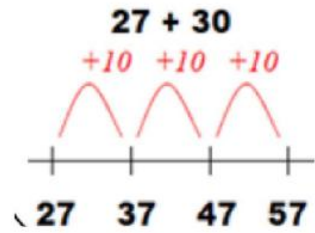
|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

$27 + 30 =$

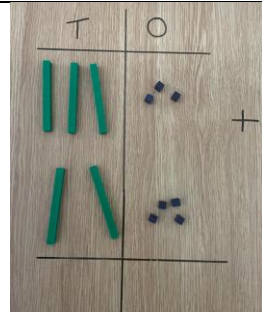
Model how to 'count on' in tens using a hundred square.  
Move on to using a number line –

$27 + 10 = 37$   
 $27 + 20 = 47$   
 $27 + 30 = 57$

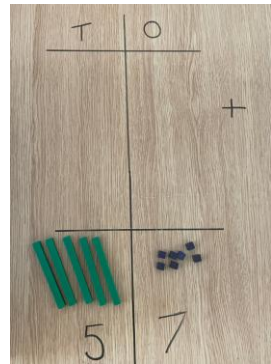
Encourage the children to spot the pattern.



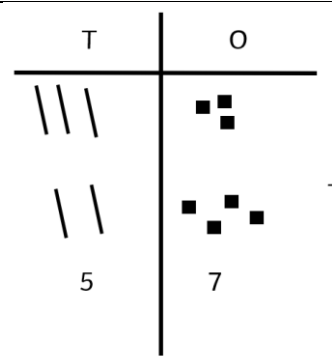
two two-digit numbers



Represent the numbers using the dienes.

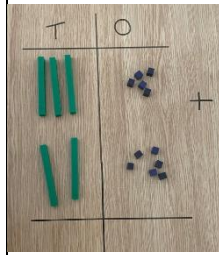


Combine the ones and write the total in the correct column. Combine the tens and write the total in the correct column.

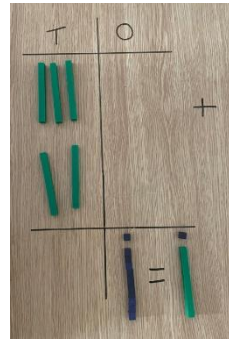


|   |       |       |
|---|-------|-------|
|   | T     | O     |
|   | 3     | 3     |
| + | 2     | 4     |
|   | <hr/> | <hr/> |
|   | 5     | 7     |

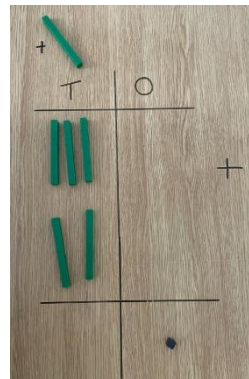
Adding 2-digit numbers (including exchange)



Set the calculation out by representing each number with the dienes.

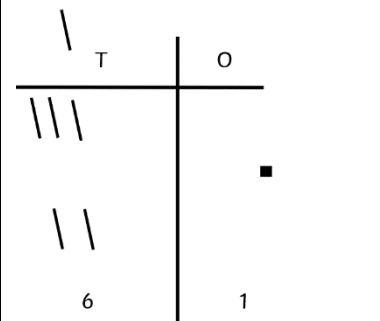
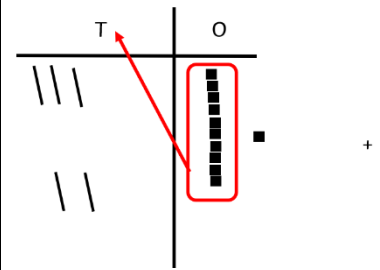
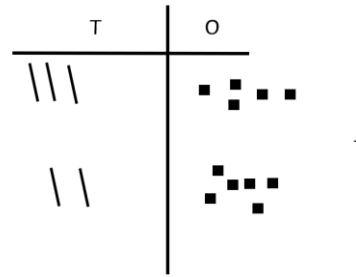


Children will need to be explicitly shown that ten ones is equal to one ten. If when combining the ones, the total reaches or exceeds ten, they **MUST exchange** ten ones for one ten. They must also write the remaining ones (if there are any) in the ones' column. In this case, we exchanged 11 ones for one ten and we will have 1 one left.



Highlight that the ten we have made is **regrouped**. This means it is placed into the tens column.

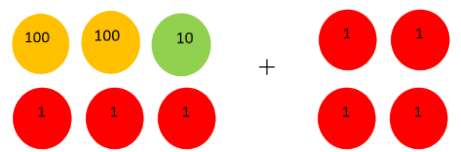
This strategy can be replicated in pictorial form.



|   | H | T  | O |
|---|---|----|---|
|   |   | +1 |   |
|   |   | 3  | 5 |
| + |   | 2  | 6 |
|   |   | 6  | 1 |



Adding 3-digit numbers and ones



Children should represent the 3-digit number. This could be with dienes or counters.

$213 + 4 =$

Encourage children to recall their number bonds to help.

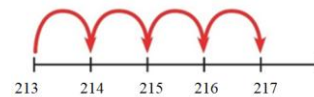
We have 3 ones and 4 ones, which makes 7 ones

So the answer will be 217

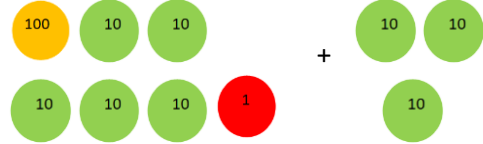
|   |   |         |
|---|---|---------|
| H | T | O       |
| ■ |   | ■ ■     |
| ■ |   | ■ ■ ■ ■ |
| 2 | 1 | 7       |

+

Children should recognise that they are 'counting on' in ones. If they have 3 ones and add 4 ones, they will have 7 ones.



Adding 3-digit numbers and tens



$151 + 30 =$

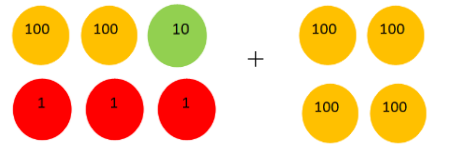
Encourage the children to look for number bond pairs.

In this case,  $5 + 3 = 8$

|           |   |   |
|-----------|---|---|
| H         | T | O |
| ■ ■ ■ ■ ■ |   | ■ |
|           |   |   |

Model the stem sentence-  
I know  $5 + 3 = 8$  so 5 tens + 3 tens = 8 tens-  $50 + 30 = 80$   
Therefore,  $151 + 30 = 181$

Adding 3-digit numbers and hundreds

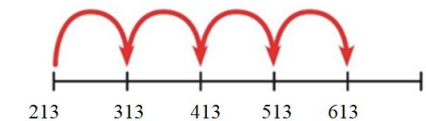


$213 + 400 = 613$

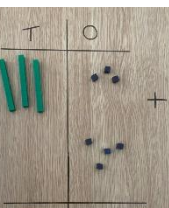
|             |   |         |
|-------------|---|---------|
| H           | T | O       |
| ■ ■ ■ ■ ■ ■ |   | ■ ■     |
| ■ ■ ■ ■ ■ ■ |   | ■ ■ ■ ■ |
| 6           | 1 | 3       |

+

Highlight that we are counting on in hundreds.



Adding numbers with up to three digits (no exchange)



Create the numbers using the dienes. Remind the children that they need to begin with the smallest digits in value first (ones).

|       |         |
|-------|---------|
| T     | O       |
|       | ■ ■ ■   |
| ■ ■ ■ | ■ ■ ■ ■ |
| 3     | 7       |

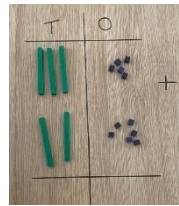
+

|   |   |   |
|---|---|---|
| 3 | 3 | + |
|   |   |   |
|   |   | 4 |
|   |   |   |
| 3 | 7 |   |



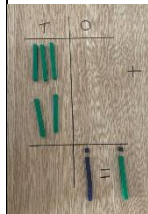
Highlight the total of the ones = 7 ones. Write the digit in the correct column. Find the total of the tens and write the number to represent the total in the correct column. Continue with the hundreds.

Adding numbers with up to three digits (with exchange)

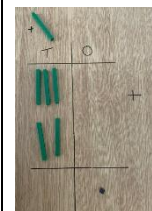


Make the addends (the numbers you are adding together) using the dienes. In the example shown, this is 35 and 26.

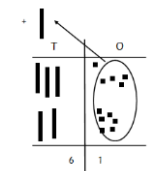
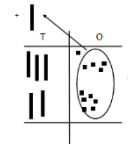
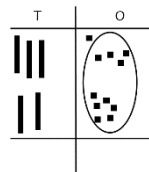
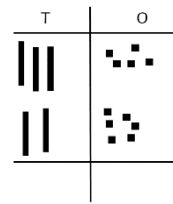
Remind the children that they must start by adding the smallest digits in value (ones). What do they notice about the total of 5 ones and 6 ones?



Show the children by combining the ones. The total is 11. 11 is made from 1 ten and 1 one. In this case we need to regroup. We will need to exchange the 11 ones for one TEN and one one.



Replace the ten ones with a ten diene.



Set the calculation out using pictorial representations to show the addends.

Calculate the total, starting with the smallest digits in value (ones).

Show the exchange of 11 ones for one ten and one one.

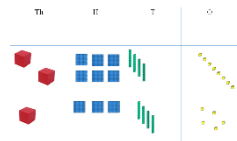
Write the totals in the correct columns.

|       |   |     |   |
|-------|---|-----|---|
|       |   |     |   |
|       |   | + 1 |   |
| 3     | 5 |     |   |
|       |   |     | + |
| 2     | 6 |     |   |
| <hr/> |   |     |   |
| 6     | 1 |     |   |
| <hr/> |   |     |   |

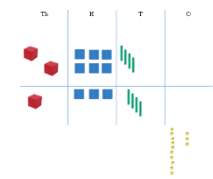
|   |  |           |          |
|---|--|-----------|----------|
|   | Complete the calculation, remembering to include the ten that was created through the regrouping.  |           |          |
| <p>Addition, add, more, and make, addend, commutative, inverse, sum, partition, total, altogether, tens boundary, hundreds boundary, <b>regroup, exchange</b></p> | <p><b><u>For Dienes:</u></b><br/> <i>We line up the ones; ___ one(s) plus ___ one(s).</i><br/> <i>We line up the tens; ___ ten(s) plus ___ ten(s).</i><br/> <b><u>For the column addition calculation:</u></b><br/> <i>The ___ is in the ones column- it represents ___ one(s); the ___ is in the ones column- it represents ___ one(s).</i><br/> <i>The ___ is in the tens column- it represents ___ ten(s); the ___ is in the tens column- it represents ___ ten(s).</i><br/> <i>In column addition, we start with the smallest digits in value.</i><br/> <i>If the column sum is equal to ten or more, we must regroup.</i></p> |           |          |
| <b><u>Year 4 -Year 6</u></b>  |  |           |          |
| Objective & Strategy  | Concrete   | Pictorial | Abstract |

Adding numbers with a range of digits, including decimals

$$2648 + 1345 =$$

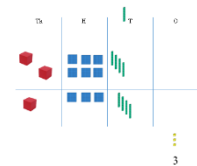


Set the calculation out using the dienes/place value counters



children should calculate the total of the ones. If a regroup is necessary (the total is more than 9) they will

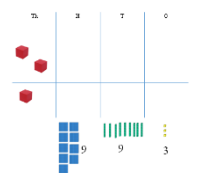
need to exchange 10 ones for a ten.



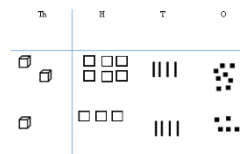
The exchange should be shown



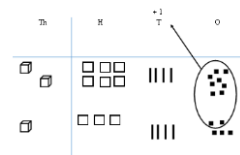
Moving on to the total of the tens, remembering to include the ten that was exchanged.



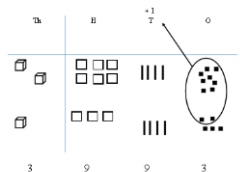
Move on to the total of the hundreds, completing exchange if needed.



Set the calculation out using pictorial representations to show the addends.

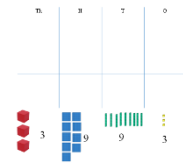


Calculate the total, starting with the smallest digits in value (the ones). If the total exceeds 9, an exchange is needed.



Write the total in the ones' column. Then complete for the tens, hundreds and thousands.

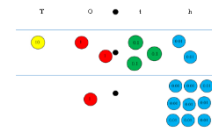
|       |   |   |     |   |  |
|-------|---|---|-----|---|--|
|       |   |   |     |   |  |
|       |   |   | + 1 |   |  |
| 2     | 6 | 4 | 8   |   |  |
| 1     | 3 | 4 | 5   | + |  |
| <hr/> |   |   |     |   |  |
| 3     | 9 | 9 | 3   |   |  |
| <hr/> |   |   |     |   |  |



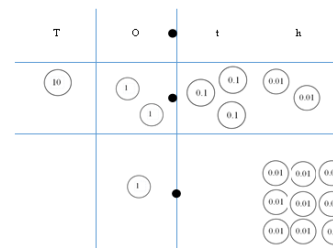
The children should continue to calculate the totals and completing the exchange process if needed.

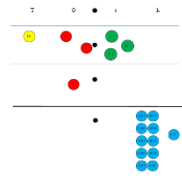
For larger numbers, place value counters can be used. The process is the same, the children will just need to use counters for ones, tens, hundreds, thousands, tens of the thousands etc. For less confident mathematicians, dienes highlight the exchange more clearly as they begin to recognise why you exchange ten ones for one ten (because ten is made of ten ones). This is not as clear when using the place value counters as the size does not change.

$$12.32 + 1.09 =$$

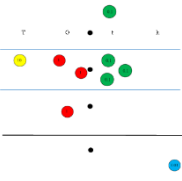


Use the place value counters to represent the addends.





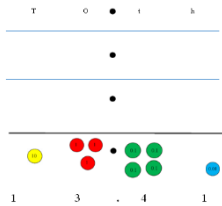
Find the total of the smallest digits in value (the hundredths). If the total exceeds 9, an exchange is required.



Show the exchange.

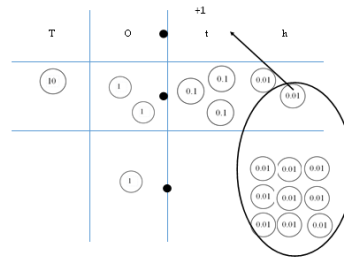


Count the total of the next smallest digits in value (tenths). Remember to include the exchanged tenth.

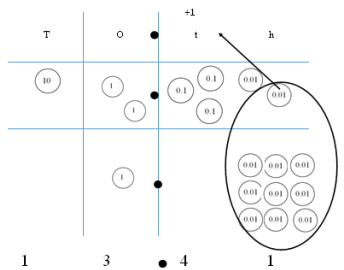


Complete the calculation, completing an regrouping required.

Rather than using the place value counters, the children should draw them.



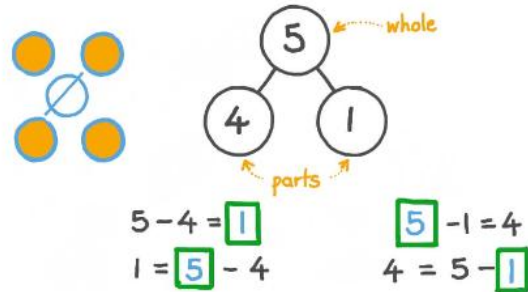
Circling the regroup of hundredths to one tenth. Show this exchange with +1 into the tenths column.



Calculate the totals, remembering to include the tenth that was creating through regrouping.

|   |   |   |     |   |   |
|---|---|---|-----|---|---|
|   |   |   |     |   |   |
|   |   |   | + 1 |   |   |
| 1 | 2 | 3 | 2   |   |   |
|   |   | 1 | 0   | 9 | + |
| 1 | 3 | 4 | 1   |   |   |
|   |   |   |     |   |   |
|   |   |   |     |   |   |

| Vocabulary  | Stem Sentences  |
|---|---|
| addition add, more, and make, addend, sum, total, commutative, inverse, altogether, tens boundary, hundreds boundary, decimal, decimal point, regroup, exchange | <p><i>Addend plus addend equals sum. Sum equals addend plus addend.</i></p> <p><i>' ___ is the whole; ___ is a part, ___ is a part, ___ is a part'</i></p> <p><i>If the column sum is equal to ten or more, we must regroup.</i></p> <p><i>The sum has increased/ decreased by ___; one addend has stayed the same, so the other addend must increase/ decrease by ___.</i></p> <p><i>' ___ tenths plus ___ tenths is ten tenths, which is equal to one whole'</i></p> <p><i>' ___ hundredths plus ___ hundredths is equal to ten hundredths, which is equal to one tenth'</i></p> <p><i>A whole split into equal parts can be seen as both an additive and a multiplicative structure'</i></p> <p><i>A whole split into unequal parts can be seen as an additive structure</i></p> |



## Subtraction

| Year 1                |          |   |          |
|-----------------------|----------|---|----------|
| Objective & Strategy  | Concrete | Pictorial   | Abstract |
| Subtraction within 20 |          | <p>Children should draw the cubes and cross out the subtrahend (the number they are subtracting).</p> |          |

Children should be taught to physically take away and remove objects (part) from the whole.



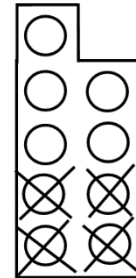
Children should be taught to use the Numicon pieces to solve subtraction calculations by selecting the piece(s) that represent the number that they are subtracting from and then covering with the piece that represents the number that they are subtracting (the children may want to use pegs to support with this process. In this instance, the children should add the correct number of pegs to show the number that they are subtracting and then add the corresponding Numicon piece.



$$5 - 3 = 2$$

$$15 - 3 = 12$$

Using a bead string to subtract 1s efficiently.

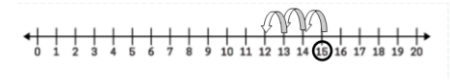


Children should draw the Numicon (or draw around the piece/pieces) and cross out the subtrahend.

Represent the number using 10 frames



Find the minuend (number you are starting with/subtracting from) and count back the correct number of places. Children could also be taught to do this mentally, counting back 2 from 5.



Finding the difference



Line up objects to find the difference between two quantities. The difference

Children should draw the object or cubes to find the difference by counting.



What is the number between? If we start at the smallest number and count on until we get to the larger number, how many jumps did we make?

between the number of blue bears and the number of red bears.

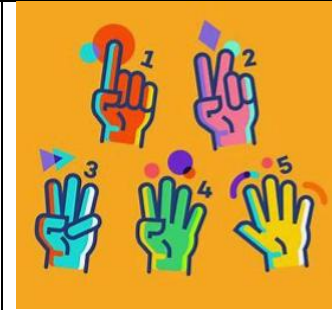
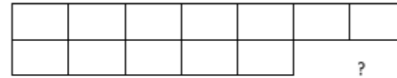


Using cubes to find the difference.



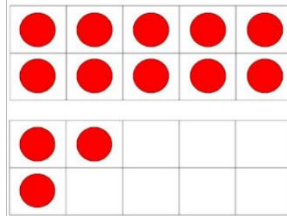
Create the number using Numicon shapes and place one part on top to find the difference between two numbers.

Using a bar model

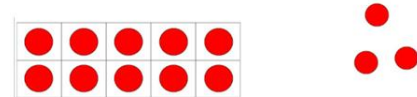


Subtracting bridging 10 using number bonds

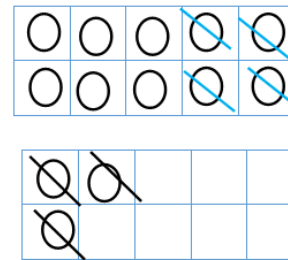
$$13 - 7 = 6$$



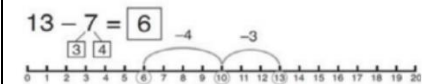
Making 13 on the tens frame. Recognising that 7 can be made from 3 and 4. If we subtract the 3, we will be left with 10.



We then need to subtract 4. We know that  $10 - 4 = 6$



Replicate the tens frames with pictorial representations.

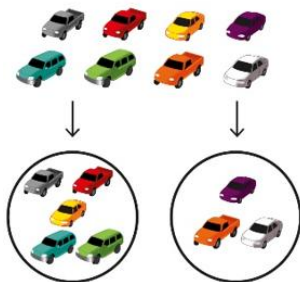


Using a number line. Partition the subtrahend. If we subtract 3, we will be left with 10.  $10 - 4 = 6$



Highlight that we know have 6 remaining. The 7 we subtracted can be counted to check.

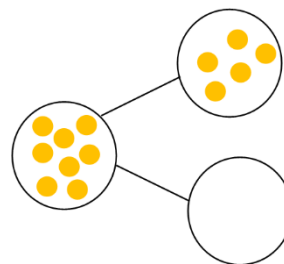
Finding a missing part when given a whole the whole and a part



$$8 - 5 = ?$$

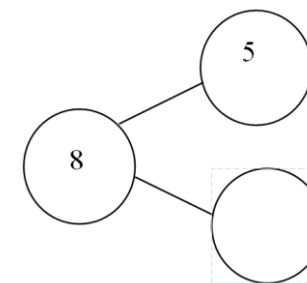
Children separate a whole into parts and understand how one part can be found by subtraction.

Children represent a whole and a part and understand how to find the missing part by subtraction.



$$8 - 5 = 3$$

Children use a part-whole model to support the subtraction to find a missing part.



Children develop an understanding of the relationship between addition and subtraction facts in a part-whole model.

$$\begin{array}{l} \square - \square = \square \\ \square - \square = \square \\ \square + \square = \square \\ \square + \square = \square \end{array}$$

**Vocabulary**

equal to, take, take-away, less, minus, subtract, leaves, distance between,

**Stem Sentences**

*One part is \_\_\_\_, the whole is \_\_\_\_. The difference is \_\_\_\_  
Subtracting one gives one less.*

difference between, how many more,  
 how many fewer/less than, most, least  
 count back, how  
 many left, how much less is...

*The more we subtract, the less we are left with. The less we subtract, the more we are left with.*

*The difference between the number of \_\_\_ and the number of \_\_\_ is \_\_\_.*




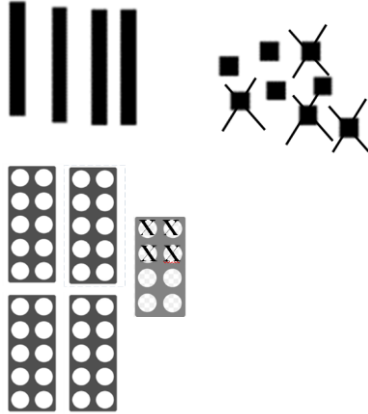
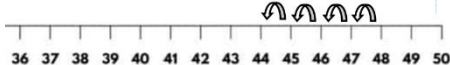
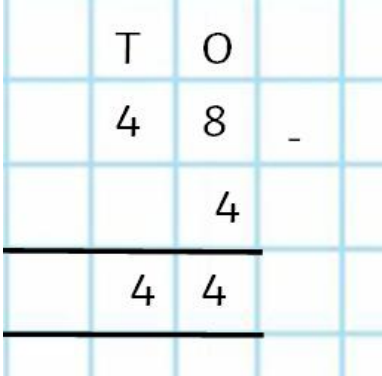
*The more we subtract, the less we are left with.*

*The less we subtract, the more we are left with.*

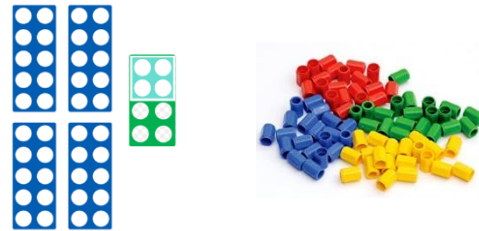
*Subtracting one gives one less*

*When zero is subtracted from a number, the number remains unchanged*

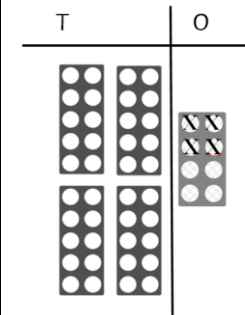
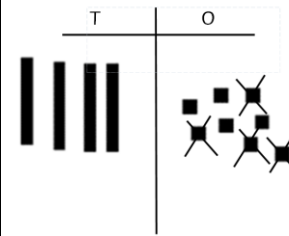
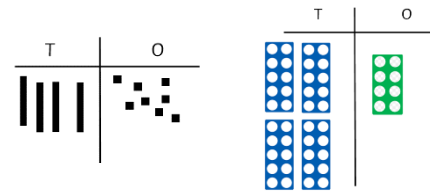
## Year 2

| Objective & Strategy                      | Concrete   | Pictorial   | Abstract  |
|---|--|---|---|
| <p>Subtract a 2-digit number and ones</p> | <p><math>48 - 4 =</math></p>  <p>The children should make the number using resources (e.g. dienes or Numicon shapes). They should then remove and 'subtract' the ones.</p>   <p>When using the Numicon shapes, the children can place the shapes that represents the subtrahend (the number they are subtracting) on top or</p> |  <p>The children should create the number using pictorial representation. They should show the subtract by crossing out individual ones.</p> <p>Again, this can be shown on a place value grid.</p> |  <p>The children should be taught to count back using a number line. Highlight the link between counting back and subtracting.</p>  <p>We know that <math>8 - 4 = 4</math><br/>       So ...<br/> <math>48 - 4 = 44</math></p> |

they can add the Numicon pegs to show the subtrahend.

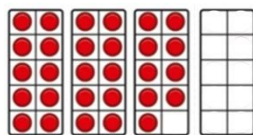
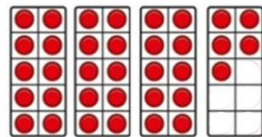


This can be shown using a place value grid to prepare the children for column method.

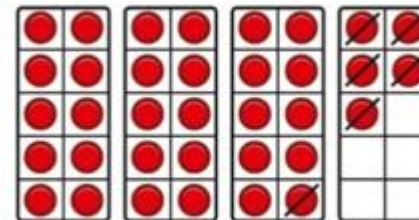


Subtracting ones when bridging is required:

$$35 - 6 =$$

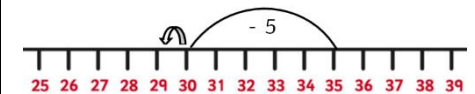


Remove the subtrahend.



Create the minuend (number you are starting with) using a pictorial representation on tens frames. First subtract the 5 you have then subtract one more from the ten.

Bridge 10 by using known bonds.



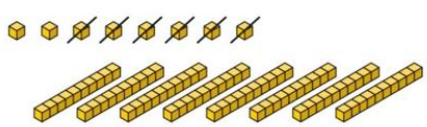
$$35 - 6 = ?$$

$$35 - 5 - 1 = 29$$

Subtract a 2-digit number and tens

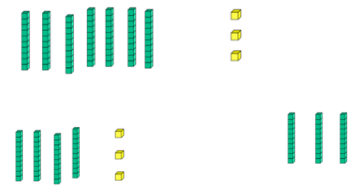
We need to take away 5 counters. Then 1 more. Children should recognise that 6 can be made from 5 and 1.

Use know number bonds and unitising to subtract multiples of 10.



8 subtract 6 is 2  
8 tens subtract 6 tens is 2 tens

$73 - 30 =$



Highlight that the ones do not change.

This can be demonstrated in a place value grid (as above).

8 subtract 6 is 2  
8 tens subtract 6 tens is 2 tens

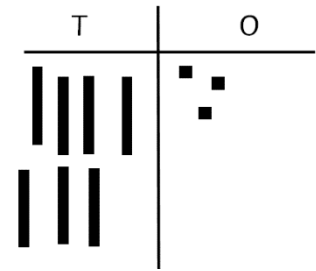
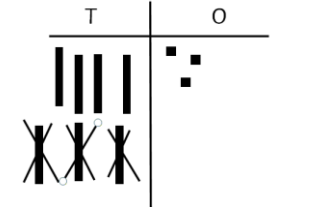
$73 - 30 =$

Highlight that the ones do not change.

This can be demonstrated in a place value grid (as above).

Subtract a 2-digit number and tens

Create the minuend (the number you are starting with) using a pictorial representation. Then cross off the tens you are subtracting- the subtrahend.

Subtract two 2-digit numbers

68 - 12 =

Create the minuend pictorially

My 1-100  
Hundred Square

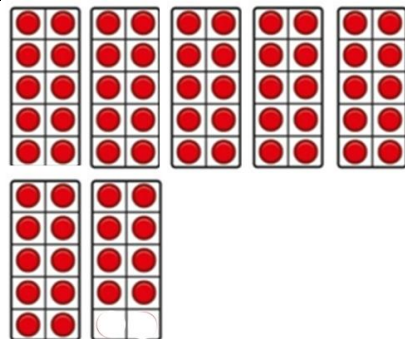
|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Using a hundred square. The children should locate the minuend and count back in tens. Highlight the pattern- only the tens column changes, the ones stay the same.

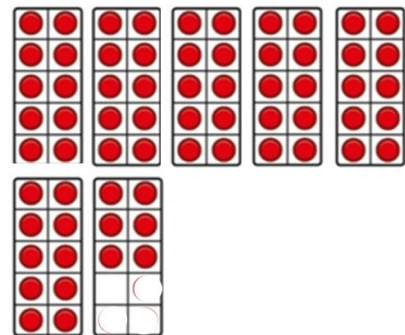
Children should move on to using the column method when appropriate.

|  |   |   |   |
|--|---|---|---|
|  | T | O |   |
|  | 7 | 3 | - |
|  | 3 | 0 |   |
|  | 4 | 3 |   |

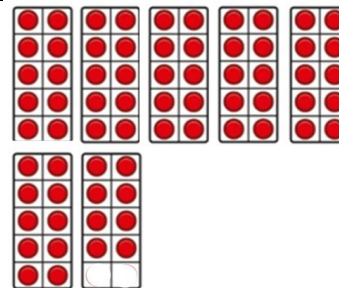
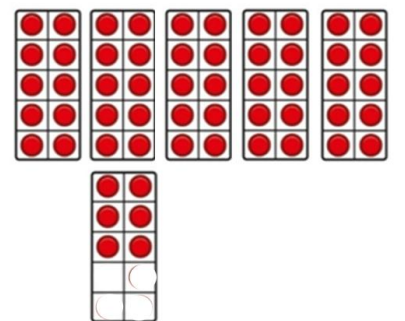
If we know that  $7 - 3 = 4$ , 7 tens subtract 3 tens will be 4 tens. Remind the children that the ones stay the same.



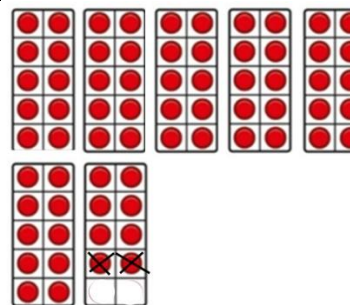
Create the number using tens frames and counters.  
First subtract the ones, then subtract the tens.



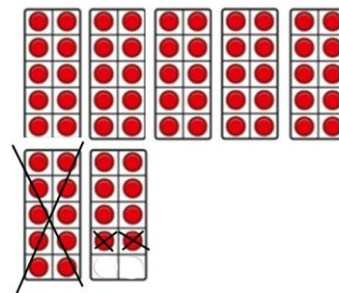
$$68 - 2 = 66$$



Subtract the ones by crossing out.



Subtract the tens by crossing out whole tens.



My 1-100  
Hundred Square

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

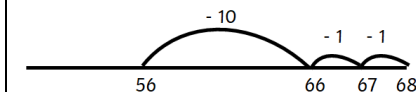
Using a hundred square. Start at the minuend. Start with the smallest digits in values (the ones). Count back.

My 1-100  
Hundred Square

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

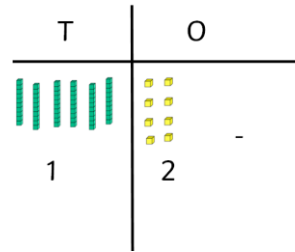
Then subtract the tens. This strategy will enable the children to bridge the tens if they are not ready for the exchange process within column method.

Using an empty number line:

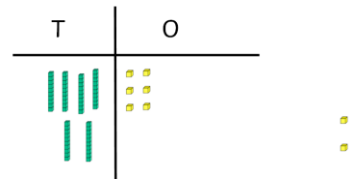


Write the minuend on the right-hand side. Subtract the ones first and write the answer below each time. Then subtract the tens. Remind the children of the

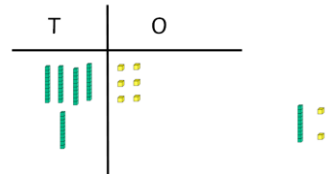
Children should recognise that is more efficient to remove one whole ten. They will be left with 5 tens and 6 ones = 56



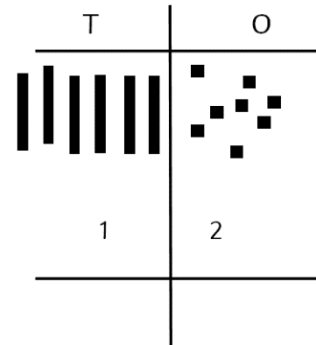
Create the minuend using dienes on a place value chart. The children should NOT create the subtrahend as this can lead to the children combining the numbers.



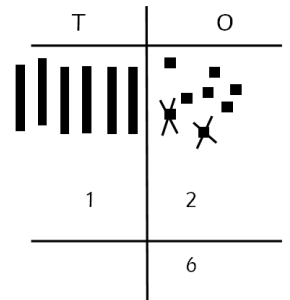
Always start by subtracting the smallest digits in value- the ones.



Then subtract the next smallest – the tens.

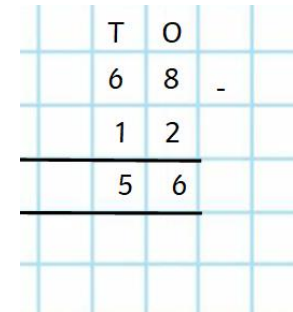


Children should create the minuend ONLY using pictorial representations. They should write the subtrahend below in the correct columns on the place value chart.



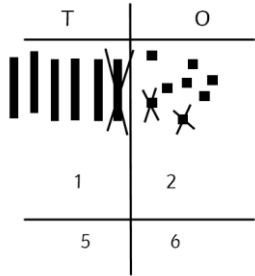
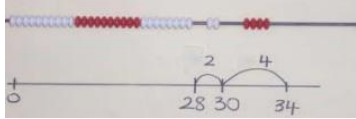
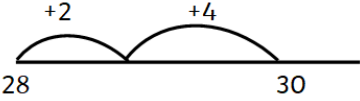
The children should cross out the correct number of ones and write the answer below (in the correct column).

pattern on the hundred square and what happens when we subtract tens- it is just the tens that change.



Children should write the minuend and the subtrahend below. Starting with the smallest digits in value (the ones) they should subtract and write the answer below.

*For children ready to learn about the exchange process, please see the Year 3 strategies.*

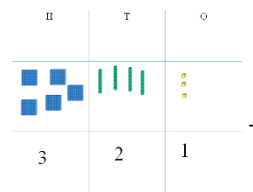
|   |   |   |  |
|---|---|---|--|
|   | <p>The children will be able to see the subtrahend (12) and the difference (56) which is their answer.</p>  |  <p>The children should cross out the correct number of tens and write the answer.</p> |  |
| <p>Subtract two 2-digit numbers:<br/>Counting on to 'find the difference'</p>   | <p><math>34 - 28 =</math></p>  <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>   |  <p>Use a number line to count on to next ten and then the rest.</p>                   | <p><math>28 + 2 = 30</math><br/><math>30 + 4 = 34</math></p> |
| <p>Vocabulary</p>   | <p>Stem Sentences</p>   |   |  |
| <p>equal to, take, take-away, less, minus, difference, count on, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is... minuend, subtrahend, place value, bridging ten</p> | <p><i>The ___ represents the number of ___ . The ___ represents the number of ___ . The ___ represents the difference between the number of ___ and the number of ___ .</i><br/> <i>To subtract ___, we can subtract the ___ then subtract the ___ .</i><br/> <i>Subtraction is not commutative</i></p> |   |  |

Year 3

|                      |          |           |          |
|----------------------|----------|-----------|----------|
| Objective & Strategy | Concrete | Pictorial | Abstract |
|----------------------|----------|-----------|----------|

Subtract numbers with up to three digits, using formal written methods of columnar subtraction

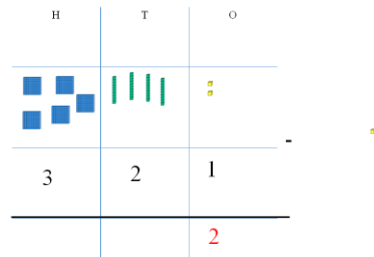
$543 - 321 =$   
Set the calculation out on a place value chart, using dienes.



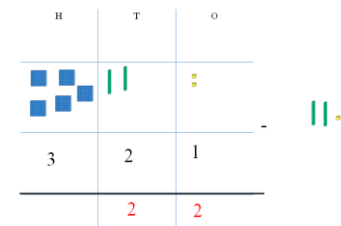
The children must only make the minuend and should write the subtrahend in digits to avoid the temptation to

combine (add).

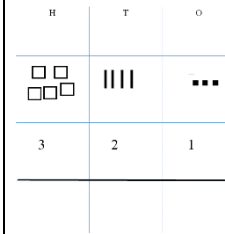
The children should begin with the smallest digits in value (ones). They should take away (subtract) the ones and write the answer in the correct column.



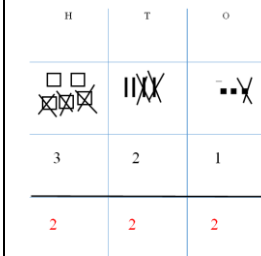
They should then move on to the next smallest digits in value (the tens).



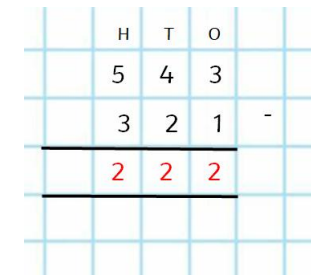
Finally, the hundreds.






Create the minuend using pictorial representations. Children **MUST** write the subtrahend and **NOT** draw it.



Starting with the smallest digits in value, cross off the representation of the subtrahend and write the difference (answer) below. Remind the children the importance of writing this in the correct column of their place value chart.



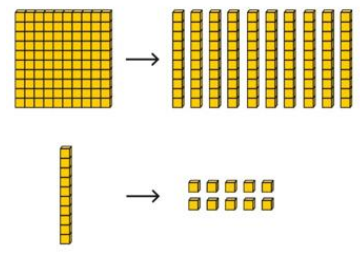
Write the minuend in the correct column of the place value chart and write the subtrahend below. Starting with the smallest digits in value, begin to subtract and write the difference (answer) below in the correct column.

| H   | T   | O   |
|---|---|---|
|  |  |  |
| 3   | 2   | 1   |
| <hr style="border: 0.5px solid black;"/>  |   |   |
| 2   | 2   | 2   |

The children should recognise the relationship between the minuend (the number they created), the subtrahend (the number they subtracted – this has been taken away and now sits outside of the place value chart) and the difference (their answer).




When exchange is required

In order for the children to fully understand the exchange process within subtraction, they need to understand the following-






1 hundred is equal to 10 tens  
1 ten is equal to 10 ones




527 - 145 =

| H   | T   | O   |
|---|---|---|
|  |  |  |
| 5   | 2   | 7   |
| <hr style="border: 0.5px solid black;"/>  |   |   |
| 1   | 4   | 5   |

Children should create the minuend and write the subtrahend in the place value chart. This could be completed on a whiteboard.

| H   | T   | O   |
|---|---|---|
|  |  |  |
| 5   | 2   | 7   |
| <hr style="border: 0.5px solid black;"/>  |   |   |
| 1   | 4   | 5   |

The children should use pictorial representations to create the minuend but **MUST** write the subtrahend.

| H  | T  | O  |
|--|--|--|
|  |  |  |
| 5  | 2  | 7  |
| <hr style="border: 0.5px solid black;"/>   |  |  |
| 1  | 4  | 5  |
| <hr style="border: 0.5px solid black;"/>   |  |  |
|  |  | 2  |

Starting with the smallest digits in value, they should begin subtracting and writing the answer in the correct column.

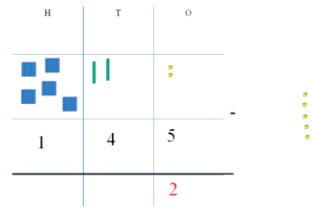
Again, they should recognise that they cannot subtract 4 tens from 2 tens so they will need to exchange.

| H  | T | O |
|--|---|---|
| 5  | 2 | 7 |
| <hr style="border: 0.5px solid black;"/> |   |   |
| 1  | 4 | 5 |
| <hr style="border: 0.5px solid black;"/> |   |   |

Children should start by setting the calculation out as they normally would and completing the first phase- subtracting the ones.

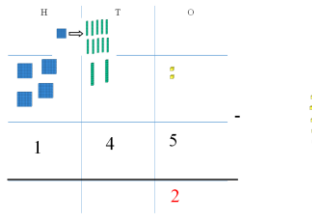
| H  | T | O |
|--|---|---|
| 5  | 2 | 7 |
| <hr style="border: 0.5px solid black;"/> |   |   |
| 1  | 4 | 5 |
| <hr style="border: 0.5px solid black;"/> |   |   |
|  |   | 2 |

Then moving on to the tens- recognising that they cannot complete this without exchanging.



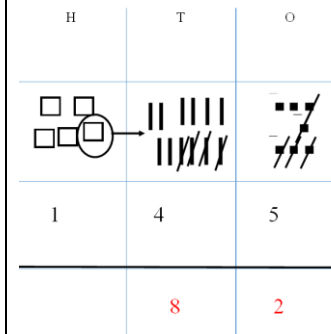
As always, the children should begin with the smallest digits in value (the ones). Removing the ones they are subtracting and writing what is left (the answer) in the correct column.

When they get to the tens, they should notice that they cannot subtract 4 tens as they only have 2 tens. Highlight that they CANNOT reverse the calculation and complete 4 tens subtract 2 tens = 2 tens! Explain that although we cannot subtract a larger from a smaller, the minuend (the number we are subtracting from *is* larger, so we need to exchange). Remind the children of the picture above showing the exchange of hundreds to tens and tens to ones.

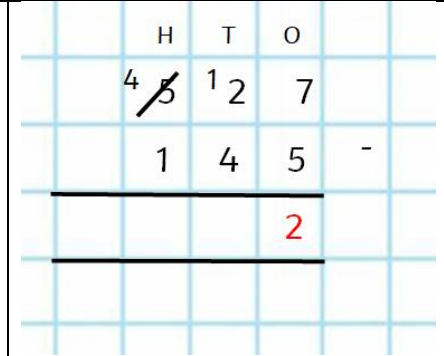
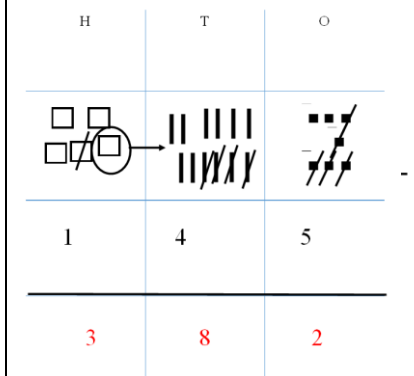


The hundred must be removed completely as we no longer have this hundred as it originally was, we have exchanged it for ten tens. So we have the original 2 tens and the 10 tens that were exchanged, so we have 12 tens.

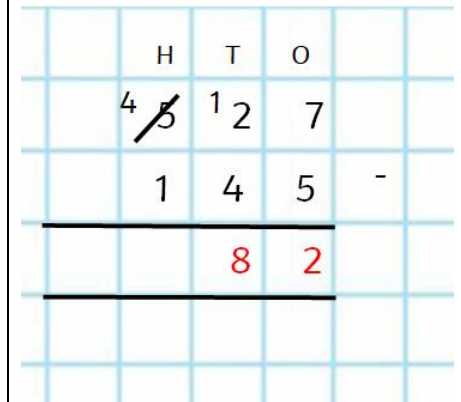
All children should be given the opportunity to complete this using concrete resources first before moving on to pictorially or using a formal written strategy to ensure that they have a secure understanding of the exchange process in subtraction.



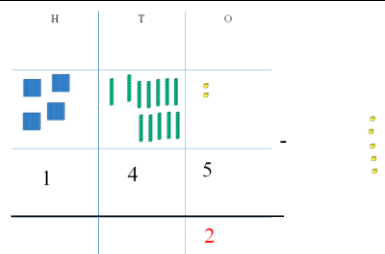
The children should show the exchange by circling the hundred and indicating where the ten tens will go. The children can also cross out the hundred after to ensure that they do not include this later on. It is a good opportunity to discuss why there are only 4 hundreds in the next phase of the calculation.



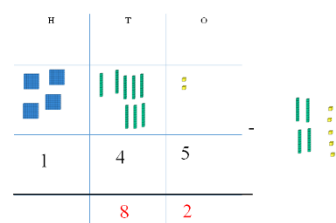
They should cross out the 5 representing the hundreds as one hundred will be exchanged for 10 tens. Writing the number of hundreds as hundreds remaining, which is 4. They should then add the 1 to the tens column to show that they now have 10 tens and the 2 originals. Highlight that by adding a 1 we create the number 12, which is the number of tens we have – 12 tens. They are then ready to complete the exchange of tens.



12 tens subtract 4 tens = 8 tens.

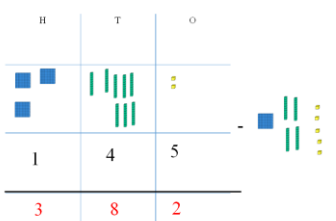


We can now complete- 12 tens subtract 4 tens.



The 4 tens should be taken away and the answer should be

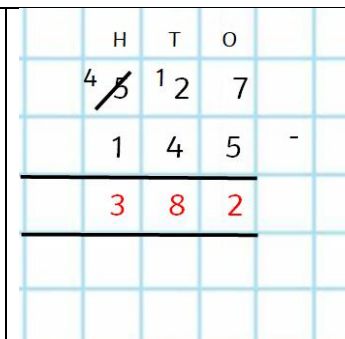
written in the correct column.



The children should finish with the hundreds, writing the answer in the correct position.

Encourage the children to check their answers by combining the subtrahend and the difference (the answer). Do the numbers add up to the minuend? This is a good opportunity to reinforce that addition is commutative and subtraction is NOT.

Now we need to subtract 1 hundred from the 4 hundred available. Children should write the answer in the correct column.



Then finally completing the subtraction of hundreds. We now have 4 hundreds, subtract 1 hundred, which gives us 3 hundreds.



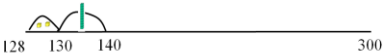
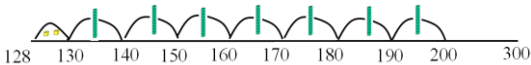
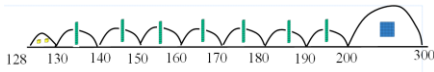
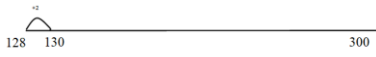
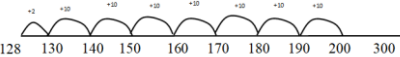
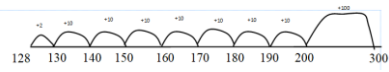
When the minuend is a multiple of 100 (subtracting from zeros)

*Using column method and exchanging is not always the most efficient method for children at this age. Children should be taught to 'count on' to find the difference (the answer) as a possible strategy for them to choose from.*

Children should follow the same process, using a blank number line. Starting at the subtrahend and counting on until they reach the minuend.

128 \_\_\_\_\_ 300

Some children will be able to do this mentally with some jottings and their number bond knowledge. If they know  $3 + 7 = 10$ , they will know that 3 tens add 7 tens = 10 tens.  $30 + 70 = 100$

|  |  |   |  |
|--|--|---|--|
|  | <p><math>300 - 128 =</math></p>  <p>128 130 300</p> <p>Explain that we will start at the subtrahend (the smaller number) and count on until we reach the minuend (the number we are subtracting from). This will give us the difference between the two numbers, which is our answer.</p>  <p>128 130 300</p> <p>They should start by adding on to the nearest ten. If they add 2 to 128, they will get to 130.</p>  <p>128 130 140 300</p> <p>They should then add on tens to reach the next hundred.</p>  <p>128 130 140 150 160 170 180 190 200 300</p> <p>When they reach the next hundred, they will need to complete jumps of 100.</p>  <p>128 130 140 150 160 170 180 190 200 300</p> <p>Children should then work out how many jumps they needed to make to calculate the difference between the 2 numbers.<br/>2 ones, 7 tens and 1 hundred</p> | <p>Counting on to the next ten first of all.</p>  <p>128 130 300</p> <p>Then counting on in tens to reach the next hundred.</p>  <p>128 130 140 150 160 170 180 190 200 300</p> <p>Finally, counting on in hundreds to reach the minuend (the number they are subtracting from).</p>  <p>128 130 140 150 160 170 180 190 200 300</p> <p>The children should then add up the difference = 172</p> | <p><math>128 + 2 = 130</math><br/> <math>130 + 70 = 200</math><br/> <math>200 + 100 = 300</math></p> |
| <b>Vocabulary</b>  | <b>Stem Sentences</b>  |   |  |
| <p>equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many</p> | <p><i>To subtract ____, we can subtract the ____ then subtract the ____.</i><br/> <i>Minuend minus subtrahend is equal to the difference</i><br/> <i>For dienes:</i><br/> <i>' ____ one(s) minus ____ one(s) is equal to ____ ones'</i></p>  |   |  |

fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, place value, minuend, subtrahend, **exchange**, inverse

*' \_\_\_ ten(s) minus \_\_\_ ten(s) is equal to \_\_\_ ten '*

Year 4 – Year 6

Objective & Strategy

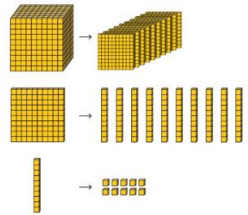
Concrete

Pictorial

Abstract

Column subtraction with exchange

Children need to understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary



2466 – 1963 =

Create the number using dienes (or place value counters).

| Th | H | T | O |
|----|---|---|---|
| 1  | 9 | 6 | 3 |

The children should NOT create the subtrahend (the number they are subtracting) with the dienes or place value counters, they should write this number digitally.

The children should draw the minuend (number they are subtracting from) using a pictorial representation.

| Th | H | T | O |
|----|---|---|---|
| 1  | 9 | 6 | 3 |

They should start with the smallest digits in values (the ones) and cross out the correct number of ones and write the answer below in the correct column.

| Th | H | T | O |
|----|---|---|---|
| 1  | 9 | 6 | 3 |
|    |   |   | 3 |

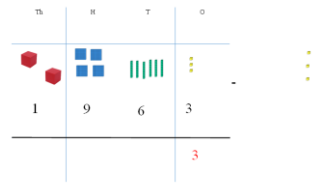
Continuing with the tens.

| Th | H | T | O |
|----|---|---|---|
| 2  | 4 | 6 | 6 |
| 1  | 9 | 6 | 3 |

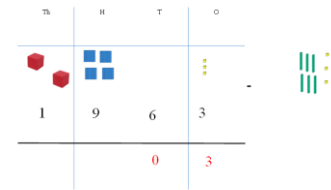
Children should start by setting the calculation out as they normally would and completing the first phase-subtracting the ones and then the tens.

| Th | H | T | O |
|----|---|---|---|
| 2  | 4 | 6 | 6 |
| 1  | 9 | 6 | 3 |
|    |   | 0 | 3 |

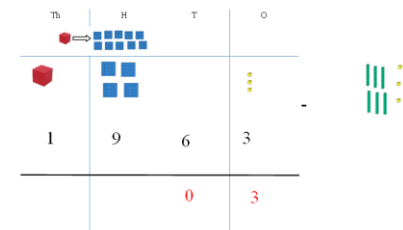
Always start with the smallest digits in value (the ones).



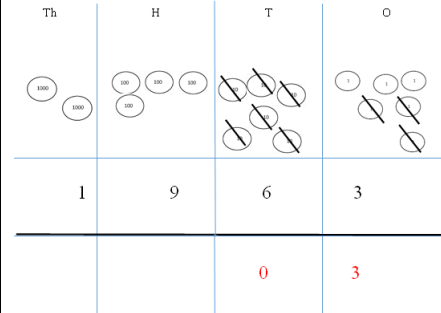
The children should physically take away the dienes and write what is left (the difference) in the correct column.



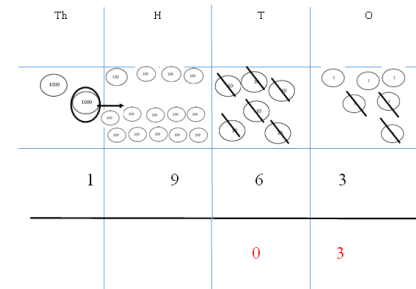
They should do the same for the next smallest digits in value (the tens).



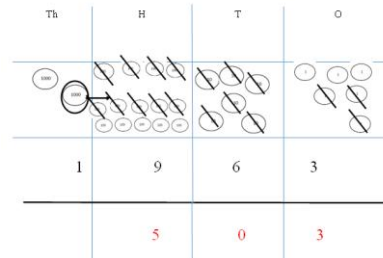
The children should recognise that they cannot subtract a larger number from a smaller number of hundreds and should realise that they need to exchange 1 thousand for 10 hundreds. The thousand must be removed completely as we no longer have this thousand as it originally was, we have exchanged it for ten



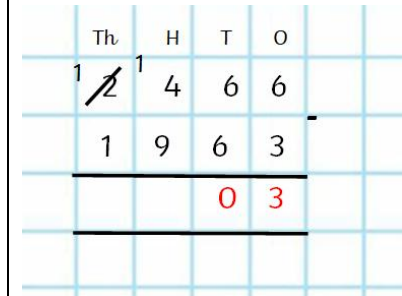
When they get to the hundreds, they should realise that they need to exchange.



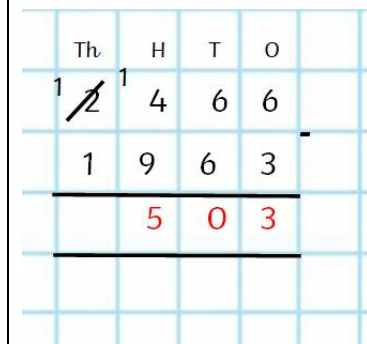
The children should show the exchange by circling the thousand and indicating where the ten hundreds will go. The children can also cross out the thousand after to ensure that they do not include this later on.



Moving on to the hundreds and recognising that they need to exchange.

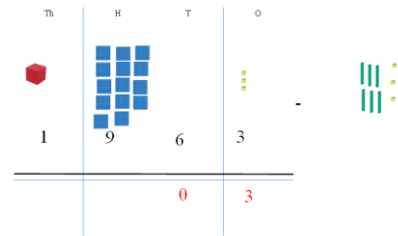


The exchange should be shown by crossing out the 2 representing the thousands as one thousand will be exchanged for 10 hundreds. Writing the number of thousands as thousands remaining, which is 1. They should then add the 1 to the hundreds' column to show that they now have 10 hundreds and the 4 originals. Highlight that by adding a 1 we create the number 12, which is the number of hundreds we have – 12 hundreds. They are then ready to complete the exchange of hundreds.

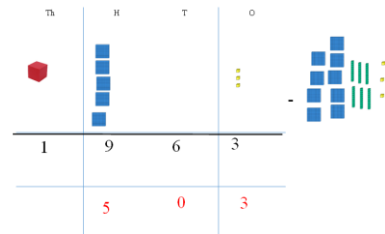


14 hundreds subtract 9 hundreds = 5 hundreds.

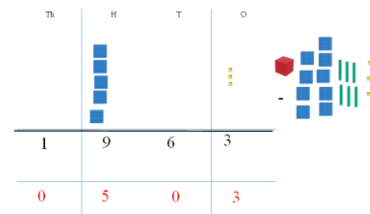
hundreds. So we have the original 4 hundreds and the 10 hundreds that were exchanged, so we have 14 hundreds.



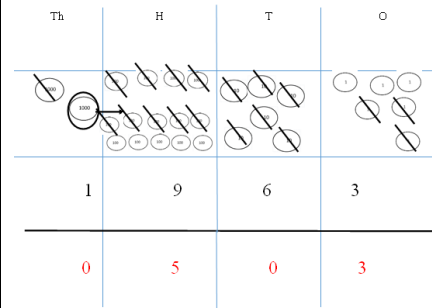
They should then complete the subtraction of hundreds and write the difference (the answer) in the correct column.



Then finally subtracting the thousands and writing the answer.



The appropriate number of hundreds should be crossed off and the answer written below.

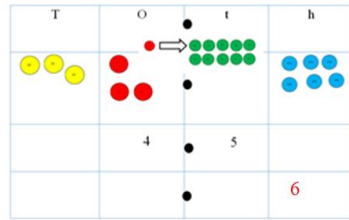


The final stage will be to subtract the thousands, by crossing off the appropriate number and writing the answer.

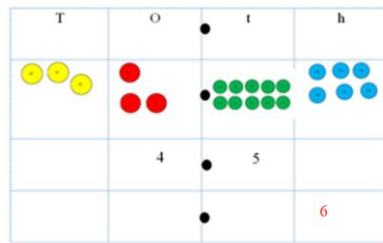
The children should complete the calculation.



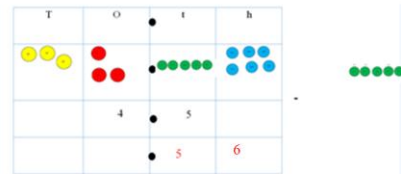




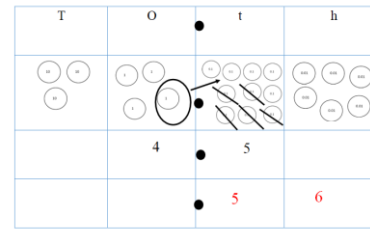
It would be a good idea to refer to the children's previous learning about exchange e.g. 1 hundred for 10 tens. In this case, they have exchanged 1 one for 10 tenths.



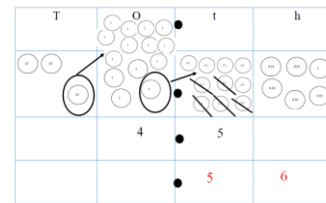
At this point, they are now ready to subtract.



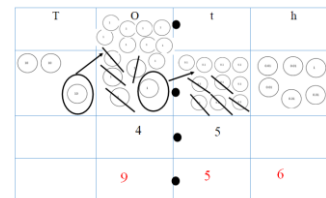
They can subtract 5 tenths from 10 tenths and this will give a difference of 5 tenths. Moving on to the next smallest digits in value (the ones) the children should recognise that they cannot subtract 4 ones from 3 ones so they will need to exchange again.



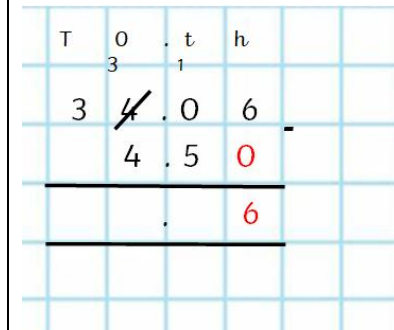
Once they have crossed out the correct number of tenths to represent the subtraction, they will be able to count how many tenths are left. They should write the answer in the correct column below.



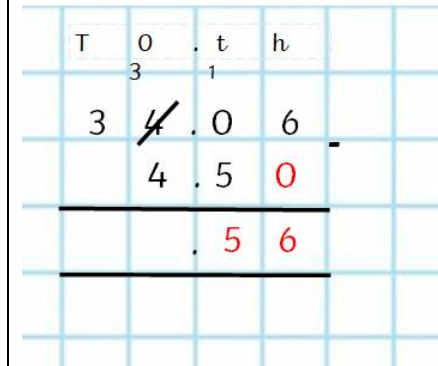
They should recognise that an exchange is needed again.



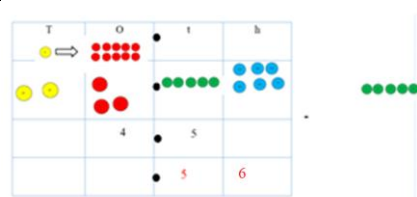
After crossing out the correct number of tenths, they should write the answer below.



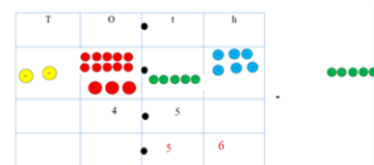
They should complete the subtraction and write the answer.



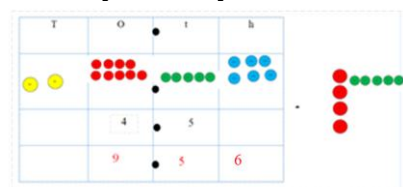
They now need to exchange again, crossing out the 3 representing the 3 tens and replacing it with 2 tens. Then adding the 1 ten above the 3 ones to show 13 ones.



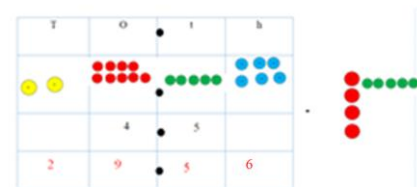
This time they are exchanging 1 ten for 10 ones.



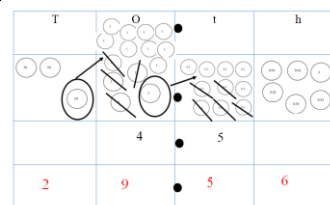
Now they are ready to subtract.



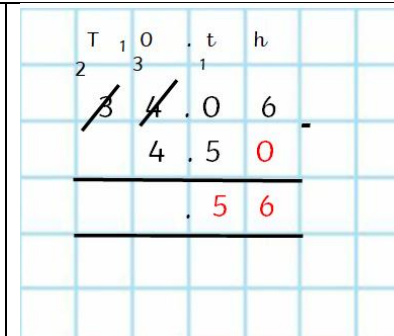
13 ones subtract 4 ones = 9 ones.



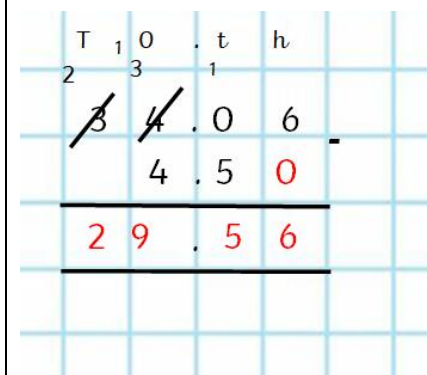
Then finally, 2 tens subtract no tens = 2 tens.



The children should complete the calculation, completing any exchange needed to fulfil the subtraction.



They can now complete the calculation and write the answers.



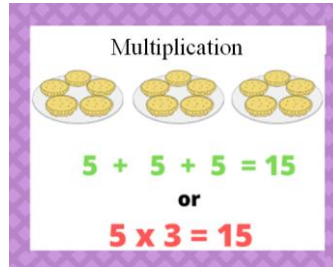
### Vocabulary


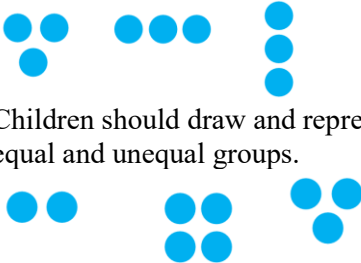

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, exchange,

### Stem Sentences

*If the minuend and the subtrahend are changed by the same amount, the difference remains the same.*  
*I've added/ subtracted \_\_\_ to/ from the minuend and the subtrahend so the difference remains the same.*  
*I've added \_\_\_ to the minuend/ subtrahend, so I need to add \_\_\_ to the subtrahend/ minuend to keep the difference the same.*  
*I've subtracted \_\_\_ from the minuend/ subtrahend so I need to subtract \_\_\_ from the subtrahend/ minuend to keep the difference the same*  
*I've added \_\_\_ to the minuend and kept the subtrahend the same, so I have to add \_\_\_ to the difference.*  
*I've subtracted \_\_\_ from the minuend and kept the subtrahend the same, so I must subtract \_\_\_ from the difference.*

|   |  |
|---|--|
| decimal, decimal point, difference, minuend, subtrahend | <p><i>'ve kept the minuend the same and added/ subtracted ___ to/ from the subtrahend so I must subtract/ add ___ to/ from the difference</i></p> <p><i>___ tenths plus ___ tenths is equal to ten tenths, which is equal to one. One is equal to ten tenths; ten tenths minus ___ tenths is equal to ___ tenths.</i></p> <p><i>___ hundredths plus ___ hundredths is equal to ten hundredths, which is equal to one tenth. One tenth is equal to ten hundredth; ten hundredths minus ___ hundredths is equal to ___ hundredths.</i></p> |
|---|--|



| Year 1                              |   |  |   |
|-------------------------------------|---|--|---|
| Objective & Strategy                | Concrete  | Pictorial  | Abstract  |
| Recognising and making equal groups |  <p>Give the children time to explore making groups. Children should arrange objects in equal and unequal groups and understand how to recognise whether they are equal or not.</p> |  <p>Children should draw and represent equal and unequal groups.</p>   | <p>Using language to describe the equal groups<br/>For example:<br/>Four equal groups of 3<br/>Three equal groups of 3</p> <p>Children should then begin to recognise they have <b>four lots of 3</b> or <b>3 four times</b>.</p> |
| Doubling                            | <p>Children should explore doubles using concrete resources.</p>  |  <p>Children could practise doubling numbers using butterfly templates or by using pictorial representations.</p> | <p>Children should begin to recognise the relationship between multiplication and addition.<br/>Double 3=<br/><math>3 + 3 = 6</math></p>  |

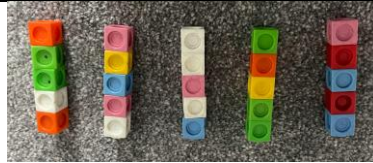


This could be using the Numicon shapes, cubes, counters or any other set of the same objects (conkers etc).



$$\text{Double } 3 = 6$$

### Counting in multiplies



Introduce the concept by showing the children the multiple grouped and practise counting in multiples.



The children could then practise counting in multiples using pictorial representations.

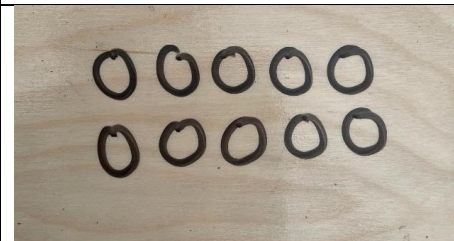


When the children are confident, they can use a number line and begin to pattern spot.

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

### Understanding arrays

Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.



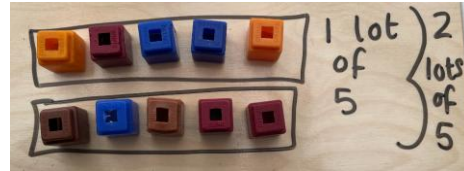
The children can represent the same array using pictorial representations.

The children should then be able to express the arrays using the multiplication symbol

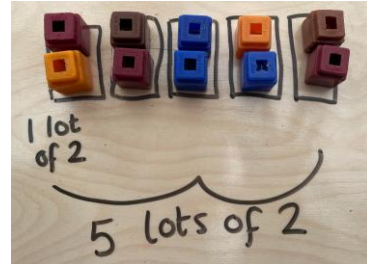
$$2 \times 5 = 10$$

$$5 \times 2 = 10$$

Showing the 2 lots of 5 by lining the cubes up into an array.



Highlighting to the children what one lot of 5 looks like and what 2 lots of 5 will look like by building it with the children.



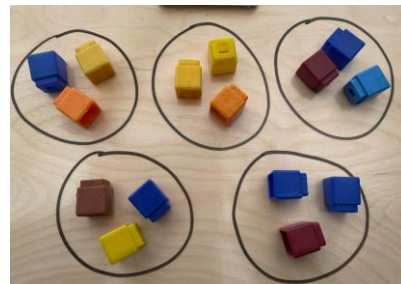
Begin to introduce the commutative nature of multiplication. Whilst we have 2 lots of 5, we are also representing 5 lots of 2.

Alternatively, the children could explore arrays using printing or stamping.

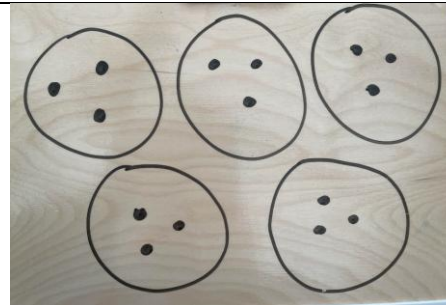


Solve one-step problems involving multiplication

Example problem:  
There are 3 sweets in one bag. How many sweets will there be in 5 bags altogether?

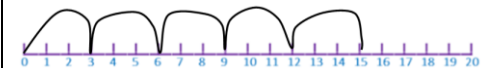


Using concrete resources to represent and solve the problem.



The children should use their own drawings to pictorially represent the problem.  
Encourage the children to think about the maths in the picture. They are 5 groups of 3. They have **3 five times or five lots of 3**. This can be show as  $3 + 3 + 3 + 3 + 3$

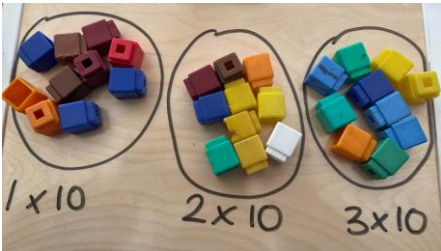
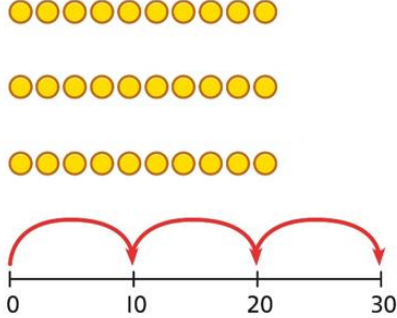
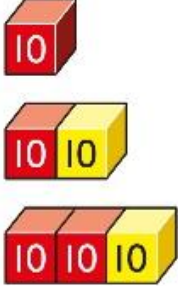
Using a number line to show the repeated addition  
 $3 + 3 + 3 + 3 + 3$

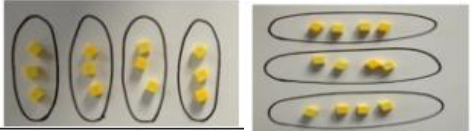
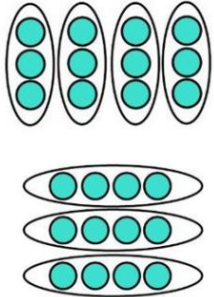
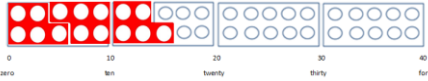
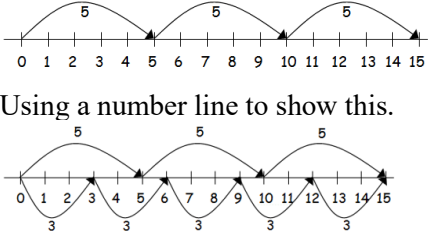
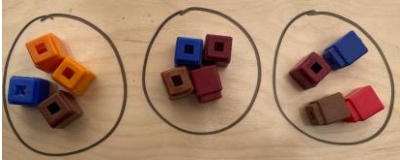



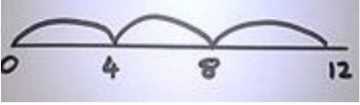


Knowing that  $5 \times 3$  is equal to  $3 \times 5$   
If the children can count in multiples of 5, they should be able to solve this mentally.

5  
10  
15


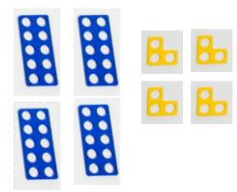

| Vocabulary   | Stem Sentences   |
|--|--|
| groups of, lots of, times, array, altogether, multiply | <p><i>The groups are equal because there are the same number in each group.</i><br/> <i>The groups are unequal because there is a different number in each group.</i></p> <p><i>One group of _____</i><br/> <i>Two group of _____</i><br/> <i>Three groups of _____</i><br/> <i>_____ group(s) of _____</i></p> <p><i>There are _____ equal groups of _____</i><br/> <i>There are _____ in each group.</i><br/> <i>There are _____ groups of _____</i></p> |

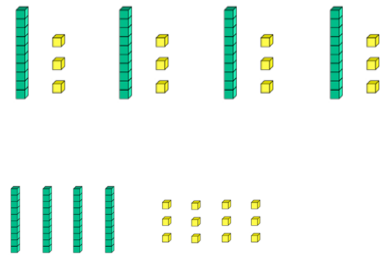
| Year 2   |  |  |   |
|--|--|--|---|
| Objective & Strategy   | Concrete   | Pictorial  | Abstract  |
| Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables | <p>Exploring this using concrete resources by creating groups of the multiple to aid counting.</p>  |  <p>Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.</p> |  <p>Understand how the times-tables increase and contain patterns.</p> |

|  |   |  |  |
|--|---|--|--|
| <p>Show that multiplication of 2 numbers can be done in any order (commutative)</p>  |  <p>Children should understand that an array can represent different calculations and as multiplication is commutative, the order of the multiplication will not affect the answer.</p>   |  <p>The children can explore this themselves, using pictorial representations.</p>  | <p>The children should then use the array to write the calculation-</p> $4 \times 3 = 12$ $3 \times 4 = 12$ <p>This is a good opportunity to reinforce multiplication as repeated addition</p> $3 + 3 + 3 + 3 = 12$ $4 + 4 + 4 = 12$ |
| <p>Using repeated addition</p>   |  <p>Using the Numicon shapes to represent 3 lots of 5 or 3 multiplied by 5 highlighting that this is the same as <math>5 + 5 + 5</math></p>   |  <p>Using a number line to show this.</p> <p>This is a good opportunity to reinforce the commutative nature of multiplication</p>   | <p>Using repeated addition</p> $5 + 5 + 5 =$ <p>Knowing that they can count in 5s to find the total</p>  |
| <p>Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts</p> | <p>Michael has 3 bags of sweets. Each bag has 4 sweets inside. How many sweets does Michael have altogether?</p>  <p>The children should use concrete resources to represent the problem and find the answer. The concept of multiplication as repeated addition could be reinforced.</p>  <p>Showing the same cubes from the groups represented on a number line.</p> |  <p>Using drawings to represent the problem pictorially.</p>  <p>Representing the problem using repeated addition.</p> | <p>Using a blank number line:</p>  <p>Recognising this as <math>3 \times 4 = 12</math></p>  |
| <p>Vocabulary</p>  | <p><b>Stem Sentences</b></p>  |  |  |

|   |  |
|---|--|
| <p>groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative.</p> | <p><i>There are ___ equal groups of ___</i><br/> <i>There are ___ in each group.</i><br/> <i>There are ___ groups of ___</i><br/> <i>___ group(s) of ___</i><br/> <i>___ group(s) of ___ make ___</i><br/> <b><u>For pictorial representations</u></b><br/> <i>___ represents the number of ___ (nests)</i><br/> <i>___ represents the number of ___ (eggs) in each ___ (nest)</i><br/> <b>For repeated addition</b><br/> <i>___ add ___ add ___</i><br/> <i>___ + ___ + ___</i></p> <p><i>There are ___ groups of ___</i><br/> <i>___ x ___ + ___</i><br/> <i>___ = ___ x ___</i></p> <p><i>There are ___ groups of ___. There are ___ altogether</i></p> |
|---|--|

Year 3

| Objective & Strategy                                 | Concrete   | Pictorial  | Abstract                   |
|--|--|--|----------------------------|
| <p>Multiplying 2digit number by a 1-digit number</p> | <p>13 x 4 =<br/>           In this example, we have 13 four times. The children could explore this concept using Numicon shapes or dienes.</p>  <p>The children could then partition the groups of 13 into tens and ones to find the total.</p>  | <p>The children could complete this pictorially.</p>  | <p>13 + 13 + 13 + 13 =</p> |



|   |    |   |
|---|----|---|
| x | 10 | 3 |
| 4 |    |   |

It is vital that the children have a secure understanding of place value and are able to partition a 2-digit number into tens and ones.

Introduce the concept using a familiar pattern using arrays. They are going to multiply the 4 by 10 and then by 3 using arrays to begin with.

|   |   |   |
|---|---|---|
| x | T | U |
|   |   |   |

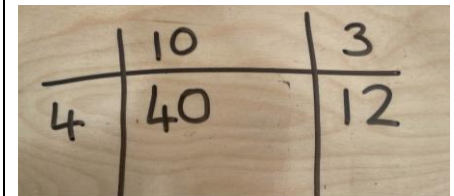
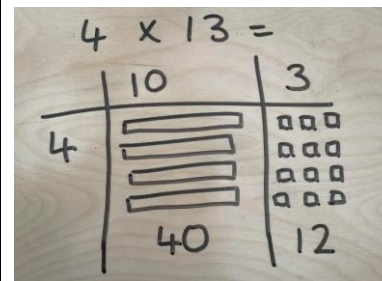
multiplication array.

Showing 4 rows of 13

They can then move on to using the dienes to represent the

It would not be an efficient strategy for the children to draw out all of the counters to represent the array so wait until the children are secure with this and are able to use the dienes more securely before moving on to a pictorial strategy.

They can they use pictorial representations to multiply within the grid using dienes.



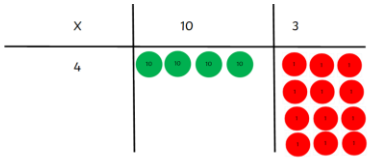
The children will need to be taught that they then need to combine the tens and ones.

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|--|--|--|--|

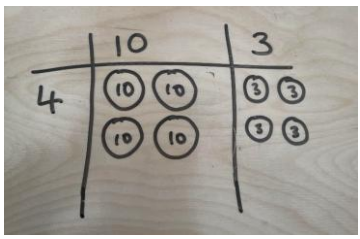
Using counters

Using counters can be a more efficient strategy to bridge the gap between using arrays and moving on to a pictorial or abstract strategy.

$13 \times 4 =$



The children now have the option to unitise the counters to use a more efficient strategy.



*As above*

**Vocabulary**

groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups,

**Stem Sentences**

*If there are \_\_\_ equal groups, we can use the \_\_\_ times table.  
 \_\_\_ is a factor so we can use the \_\_\_ times table.*

times as big as, commutative, product, multiples of

Partition \_\_\_ into \_\_\_ and \_\_\_  
 Multiply the ones \_\_\_ x \_\_\_  
 Multiple the tens \_\_\_ x \_\_\_  
 \_\_\_ tens x \_\_\_ = \_\_\_ tens  
 \_\_\_ ones x \_\_\_ = \_\_\_ ones

Year 4

Objective & Strategy

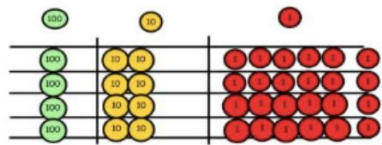
Concrete

Pictorial

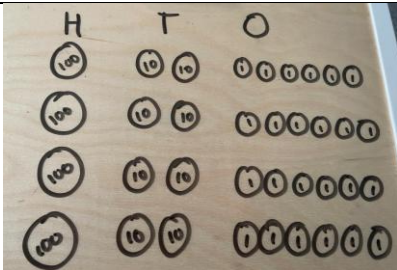
Abstract

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout  
*Also see above*

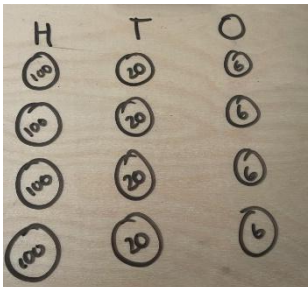
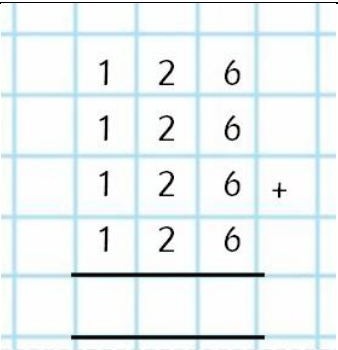
4 x 126 =



The children have made 126 four times and now need to add the 'lots of 126' together as they would using column addition.

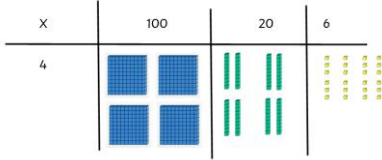
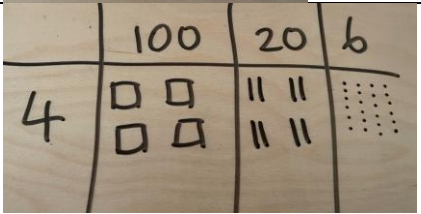


If the children feel confident counting in multiples (multiples of 20 and 6 in this example) they could draw unitising counters.

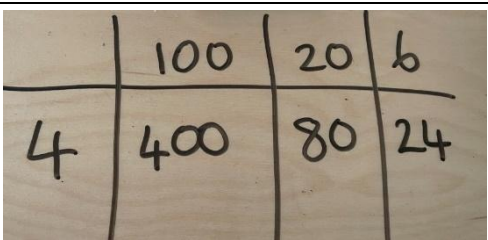



Using and applying their knowledge of multiplication as repeated addition

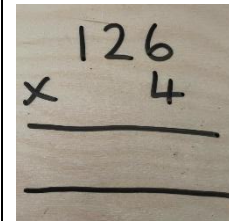
Children could use place value counters or move straight into using dienes as this is a more efficient strategy.

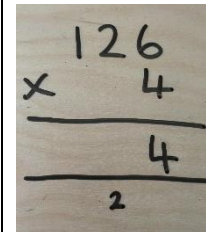
The children should use pictorial representations of the dienes.



When appropriate, the children should be taught standard multiplication as a strategy to multiply.

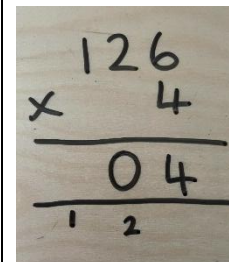

$$\begin{array}{r} 126 \\ \times 4 \\ \hline \\ \hline \end{array}$$

First of all, the children need to set the calculation out correctly.

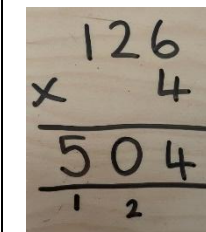

$$\begin{array}{r} 126 \\ \times 4 \\ \hline 4 \\ \hline 24 \\ \hline \end{array}$$

Then they should begin multiplying the smallest digit from the factor by the single digit factor. When writing the product, they should

write the ones in the ones' column and show the tens below the line so that this can be added to the next product made.


$$\begin{array}{r} 126 \\ \times 4 \\ \hline 04 \\ \hline 84 \\ \hline \end{array}$$

Moving on to multiplying the next smallest in value (the tens in this case). They must remember to add on the 2 tens.


$$\begin{array}{r} 126 \\ \times 4 \\ \hline 504 \\ \hline \end{array}$$

The final step has been completed.

Vocabulary

Stem Sentences

groups of, lots of, times, array,  
altogether, multiply, multiplied by,

\_\_\_\_\_ is a factor

repeated addition, sets of, equal groups,  
times as big as, commutative, product,  
factor, multiples of, scale  
up, inverse, derive

\_\_\_\_\_ is a factor  
The product of \_\_\_ and \_\_\_ is \_\_\_  
\_\_\_\_\_ is the product of \_\_\_ and \_\_\_

If there are \_\_\_ equal groups, we can use the \_\_\_ times table.  
\_\_\_ is a factor so we can use the \_\_\_ times table.

Partition \_\_\_ into \_\_\_ and \_\_\_  
Multiply the ones \_\_\_ x \_\_\_  
Multiple the tens \_\_\_ x \_\_\_

\_\_\_ hundreds x \_\_\_ = \_\_\_ hundreds  
\_\_\_ tens x \_\_\_ = \_\_\_ tens  
\_\_\_ ones x \_\_\_ = \_\_\_ ones

Partition \_\_\_ into \_\_\_ and \_\_\_  
\_\_\_ x \_\_\_ ones = \_\_\_ ones

Write the \_\_\_ in the ones column (and \_\_\_ in the tens column)

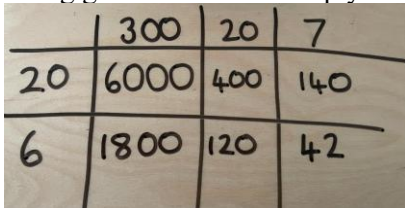
\_\_\_ x \_\_\_ tens = \_\_\_ tens

Write the \_\_\_ in the tens column (and \_\_\_ in the hundreds column)

\_\_\_ x \_\_\_ ones = \_\_\_ ones + \_\_\_ tens  
\_\_\_ x \_\_\_ tens = \_\_\_ tens + \_\_\_ hundreds.

\_\_\_\_\_ s are \_\_\_\_\_ (writing down \_\_\_ below the tens column and \_\_\_ in the ones column.)  
\_\_\_\_\_ s are \_\_\_\_\_, plus \_\_\_\_\_ is \_\_\_\_\_ (write down \_\_\_ below the hundred column and + \_\_\_ in the tens column)  
\_\_\_\_\_ s are \_\_\_\_\_; plus \_\_\_\_\_ is \_\_\_\_\_ (writing down \_\_\_\_\_ in the thousands column and \_\_\_\_\_ in the hundreds column)

### Year 5 & 6

| Objective & Strategy  | Concrete | Pictorial  | Abstract   |
|---|----------|--|--|
| Multiply numbers up to 4 digits by a one- or two-digit number |          | Using grid method to multiply<br> | To begin with, show the children each step separately so that they understand the process- |

The children should then add the products together.

$$\begin{array}{r} 327 \\ \times 26 \\ \hline \end{array}$$

$$\begin{array}{r} 327 \\ \times 26 \\ \hline 1962 \\ 6540 \\ \hline 21382 \end{array}$$

It is important for the children to recognise the place value of each digit. This will support

the concept of using zero as a place holder later on.

They should then add the numbers together to find the product of 327 and 26.

The more efficient method-

$$\begin{array}{r} 327 \\ \times 26 \\ \hline \end{array}$$

$$\begin{array}{r} 327 \\ \times 26 \\ \hline 1962 \\ \hline 6540 \\ \hline 21382 \end{array}$$

Starting off by multiplying the factor by the ones in the second factor

$$\begin{array}{r} 327 \\ \times 26 \\ \hline 1962 \quad 327 \times 6 \\ 6540 \quad 327 \times 20 \\ \hline 8502 \end{array}$$

Following the process as they would use when multiplying by a single digit factor.

$$\begin{array}{r} 327 \\ \times 26 \\ \hline 1962 \quad 327 \times 6 \\ 6540 \quad 327 \times 20 \\ \hline 8502 \end{array}$$

When multiplying the tens, the children MUST remember to add a zero as a place holder. This is because (in this example) they are not multiplying 2 by 7, they are multiplying 20 (or 2 tens) by 7.

$$\begin{array}{r} 327 \\ \times 26 \\ \hline 1962 \quad 327 \times 6 \\ 6540 \quad 327 \times 20 \\ \hline 8502 \end{array}$$

0 as a place holder

$$\begin{array}{r} 327 \\ \times 26 \\ \hline 1962 \quad 327 \times 6 \\ 6540 \quad 327 \times 20 \\ \hline 8502 \end{array}$$

Again, the children should follow the process through.

$$\begin{array}{r} 327 \\ \times 26 \\ \hline 1962 \quad 327 \times 6 \\ 540 \quad 327 \times 20 \\ \hline \end{array}$$

$$\begin{array}{r} 327 \\ \times 26 \\ \hline 1962 \quad 327 \times 6 \\ 6540 \quad 327 \times 20 \\ \hline \end{array}$$

Until they have multiplied each digit in the first factor by every digit in the second factor.

$$\begin{array}{r} 327 \\ \times 26 \\ \hline 1962 \quad 327 \times 6 \\ + 6540 \quad 327 \times 20 \\ \hline 8502 \end{array}$$

The final step is for the children to add the two products together. They should understand that this number is the product of the factors (in this example, 8502 is the product of 327 and 26).

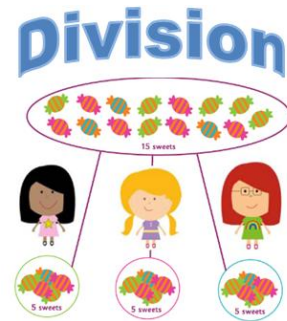
**Vocabulary**

**Stem Sentences**




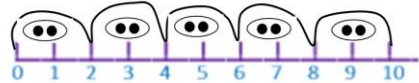
groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product,

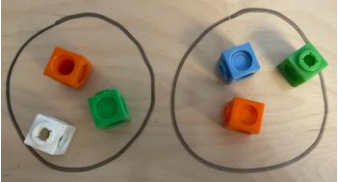
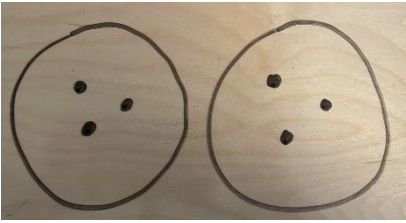
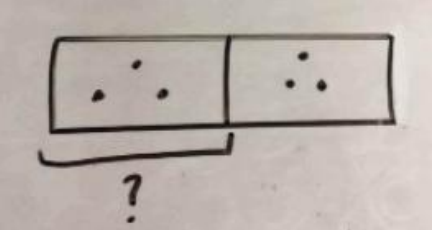
\_\_\_\_\_s are \_\_\_\_\_ (writing down \_\_\_\_\_ below the tens column and \_\_\_\_\_ in the ones column.)  
 \_\_\_\_\_s are \_\_\_\_\_, plus \_\_\_\_\_ is \_\_\_\_\_ (write down \_\_\_\_\_ below the hundred column and + \_\_\_\_\_ in the tens column)  
 \_\_\_\_\_s are \_\_\_\_\_; plus \_\_\_\_\_ is \_\_\_\_\_ (writing down \_\_\_\_\_ in the thousands column and \_\_\_\_\_ in the hundreds column)

multiples of, scale up, inverse, derive,  
factor pairs, composite numbers, prime  
number, factors, squared, cubed,



### Year 1

| Objective & Strategy                    | Concrete  | Pictorial  | Abstract   |
|---|---|--|--|
| <p>Understanding and using grouping</p> | <p>Learning to make equal groups from a whole and find how many equal groups of a certain size can be made</p>  <p>Sort a whole set of objects into equal groups.</p>  <p>There are 10 conkers altogether<br/>There are 2 in each group<br/>There are 5 group</p> |  <p>Represent a whole and work out how many equal groups.</p> <p>There are 10 in total.<br/>There are 2 in each group.<br/>There are 5 groups.</p> |  <p>Children may relate this to counting back in steps of 2, 5 or 10.</p> |

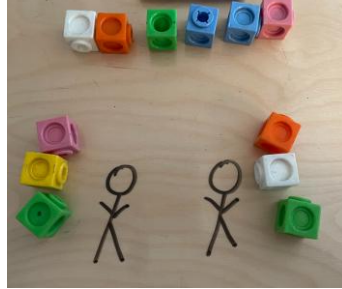
|                |   |  |   |   |   |
|----------------|---|--|---|---|---|
| <p>Sharing</p> | <p>6 shared by 2<br/> <math>6 \div 2 =</math></p>  | <p>Children should then share using pictorial representations.</p>   | <p><math>6 \div 2 = 3</math></p> <table border="1" data-bbox="1653 178 2056 242"> <tr> <td>3</td> <td>3</td> </tr> </table> <p>The children should also be encouraged to use their times table facts.</p> | 3 | 3 |
| 3              | 3   |  |   |   |   |

|  |   |
|--|---|
| <p>Vocabulary</p>  | <p><b>Stem Sentences</b></p>  |
| <p>share, share equally, one each, two each..., group, groups of, lots of, array</p> | <p> <math>\underline{\hspace{2cm}}</math> <i>divided into groups of</i> <math>\underline{\hspace{2cm}}</math><br/> <i>There are</i> <math>\underline{\hspace{2cm}}</math> <i>groups of</i> <math>\underline{\hspace{2cm}}</math>; <i>there are</i> <math>\underline{\hspace{2cm}}</math> <i>altogether.</i><br/> <math>\underline{\hspace{2cm}}</math> <i>is divided into groups of</i> <math>\underline{\hspace{2cm}}</math>. <i>There are</i> <math>\underline{\hspace{2cm}}</math> <i>groups.</i><br/> <math>\underline{\hspace{2cm}}</math> <i>is divided into</i> <math>\underline{\hspace{2cm}}</math> <i>groups of</i> <math>\underline{\hspace{2cm}}</math> </p> <p> <math>\underline{\hspace{2cm}}</math> <i>are shared equally between</i> <math>\underline{\hspace{2cm}}</math>. <i>Each child gets</i> <math>\underline{\hspace{2cm}}</math><br/> <math>\underline{\hspace{2cm}}</math> <i>divided between</i> <math>\underline{\hspace{2cm}}</math> <i>is equal to</i> <math>\underline{\hspace{2cm}}</math> <i>each.</i> </p> |

| Year 2               |          |           |          |
|----------------------|----------|-----------|----------|
| Objective & Strategy | Concrete | Pictorial | Abstract |

Sharing equally

Start with a whole and share into equal parts, one at a time.



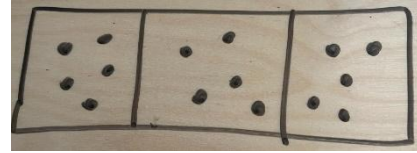
12 shared equally between 2.  
They get 6 each.



Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to each person. Keep going until all the objects have been shared

15 shared equally between 3.  
They get 5 each.

Represent the objects shared into equal parts pictorially, using a bar model.



15 shared into 3 equal parts.  
There are 5 in each part.

|   |   |   |
|---|---|---|
| 5 | 5 | 5 |
|---|---|---|

$$15 \div 3 = 5$$

Grouping equally

Understand how to make equal groups from a whole.

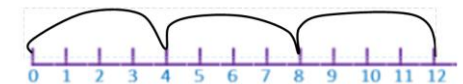


12 divided into 3 equal groups.  
There are 4 in each group.

Grouping using pictorial representations.



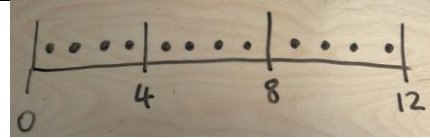
Children should begin to explore using a number line-



Highlight how to relate division by grouping to repeated subtraction.

Using their times tables when dividing with 2, 5 and 10.

Highlight the relationship between grouping and the division statements.



- 1 x 5 = 5
- 2 x 5 = 10
- 3 x 5 = 15
- 4 x 5 = 20**
- 5 x 5 = 25
- 6 x 5 = 30
- 7 x 5 = 35
- 8 x 5 = 40
- 9 x 5 = 45
- 10 x 5 = 50

I can use my 5 times tables to help me.

I know that 4 x 5 is 20

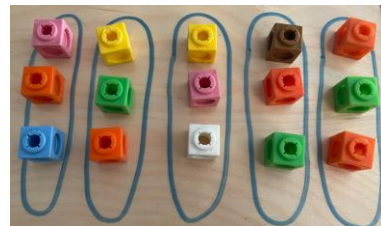
I know that 4 groups of 5 makes 20, so I know that 20 divided by 5 is 4.

$$4 \times 5 = 20 \quad \text{so} \quad 20 \div 5 = 4$$

Using and understanding arrays

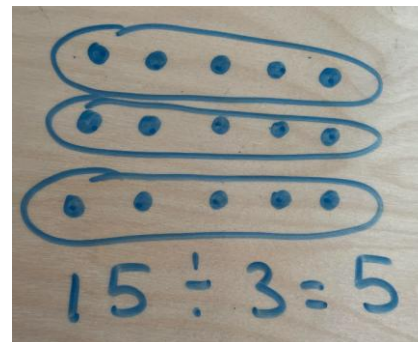
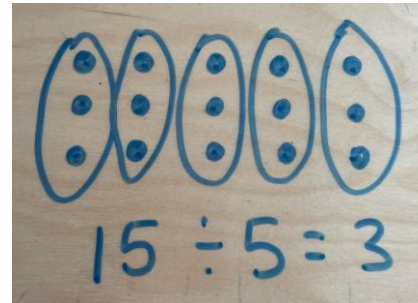


Organising the whole into groups of 5

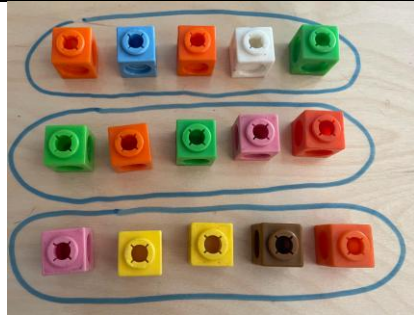


There are 5 groups and 3 in each group.

The children can also explore this pictorially



Using arrays should enable the children to see the link between division and their times tables.



There are 3 groups and 5 in each group.

Vocabulary

Stem Sentences

*There are \_\_\_ group of \_\_\_; there are \_\_\_ altogether*

*\_\_\_ is divided into groups of \_\_\_ There are \_\_\_ groups \_\_\_ is divided into \_\_\_ groups of \_\_\_*

*The \_\_\_ represents the total number of \_\_\_ The \_\_\_ represents the number of \_\_\_ in each group*

*\_\_\_ is the dividend \_\_\_ is the divisor \_\_\_ is the quotient*

*We can represent this as \_\_\_ divided between \_\_\_ \_\_\_ divided between \_\_\_ is equal to \_\_\_ each*

*One \_\_\_ is one each. That's \_\_\_. I have \_\_\_ left*

*Two \_\_\_ is two each. That's \_\_\_. I have \_\_\_ left*

*e.g. one five is one each. That's five*

*two fives is two each. That's ten*

Year 3

Objective & Strategy

Concrete

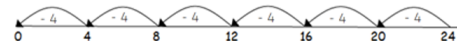
Pictorial

Abstract

Repeated subtraction

Children should make the dividend using the Numicon shapes.

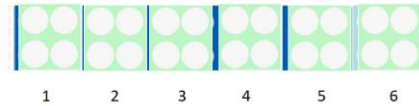
$24 \div 4 =$



Using a number line to subtract.

Applying times table knowledge. If I count in 4s, how many lots of 4 will I need to reach 24?

They should then investigate how many groups of 4 there are in the whole (24) by placing the 4 shape on top. A Numicon number line could be used to support here if needed.

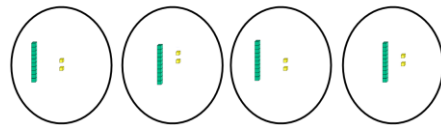
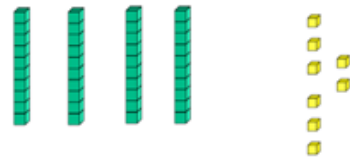


There are 6 groups of 4 in 24.  
 $24 \div 4 = 6$

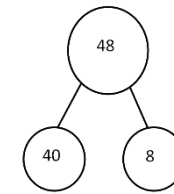
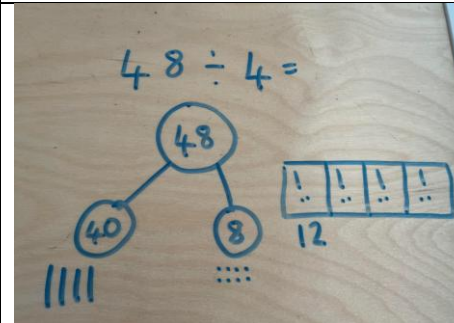
Dividing a 2-digit number by a 1-digit number

$$48 \div 4 =$$

The children should use the dienes to create the dividend.



$$48 \div 4 = 12$$



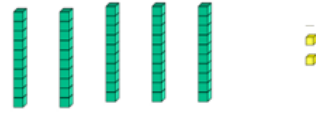
$$40 \div 4 = 10$$

$$8 \div 4 = 2$$

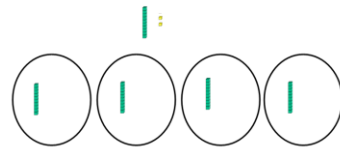
$$10 + 2 = 12$$

Dividing a 2-digit number by a 1-digit number

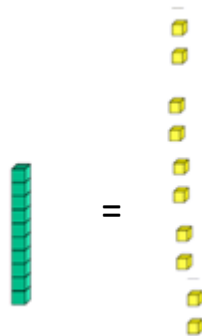
$52 \div 4 =$   
The children should make the dividend using dienes.



The children should notice quite early on that this will not be a 'perfect fit' as in the previous example.

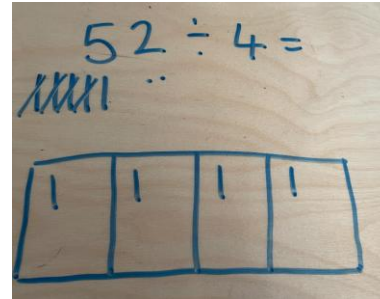


Encourage to think about the number that is remaining. Can they divide 12 by 4? How do they know? Remind the children that 1 ten is made of 10 ones.

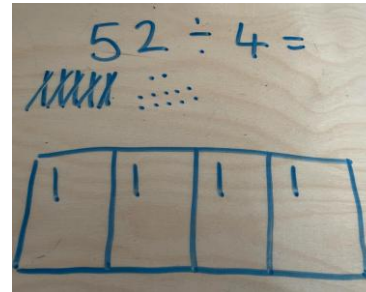


Continuing by dividing 12 ones by 4.

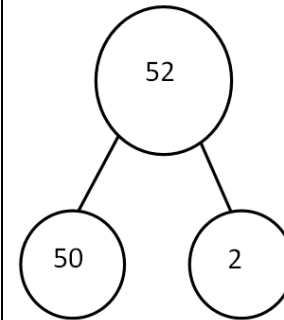
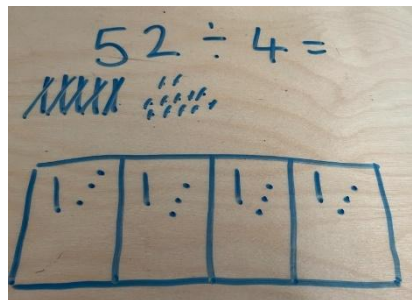
Pictorially represent the number and begin to show the division.



Complete the exchange of 1 ten for 10 ones.



This represents the calculation and shows that 52 divided by 4 is equal to 13.

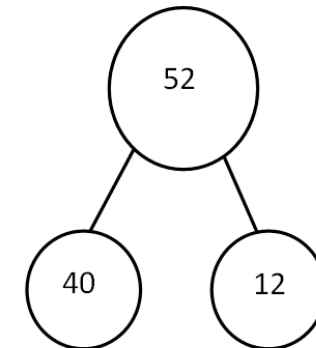


$$50 \div 4 = 10 \text{ with } 10 \text{ left over}$$

$$12 \div 4 = 3$$

$$10 + 3 = 13$$

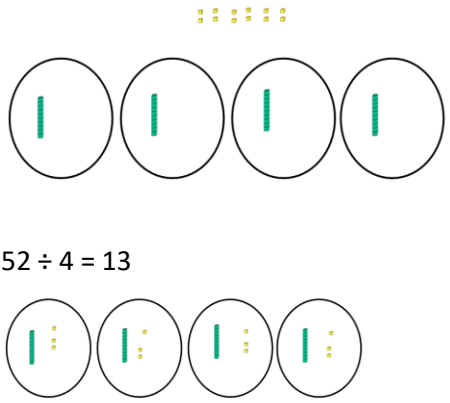
The children could also begin to apply what they have learnt about non-standard partitioning –




$$40 \div 4 = 10$$

$$12 \div 4 = 3$$

$$10 + 3 = 13$$

|   |  |  |  |
|---|--|--|--|
|   |  <p style="text-align: center;"><math>52 \div 4 = 13</math></p>  |  |  |
| Vocabulary  | <b>Stem Sentences</b>  |  |  |
| <p>Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, remainder, partitioning, non-standard partitioning</p> | <p>___ <i>tens</i> divided by ___ <i>is equal to</i> ___ <i>tens each.</i><br/>         ___ <i>ones</i> divided by ___ <i>is equal to</i> ___ <i>one each.</i><br/>         ___ <i>tens</i> and ___ <i>ones</i> make ___ <i>each</i></p> |  |  |

### Year 4

| Objective & Strategy     | Concrete  | Pictorial                                      | Abstract   |
|--------------------------|---|--|--|
| <p>Exploring factors</p> | <p>Allow the children time to explore different numbers by creating arrays or grouping the numbers into different quantities.</p>  <p>2 lots of 6 or 6 lots of 2</p> | <p>This could also be explored pictorially</p> | <p> <math>1 \times 12 = 12</math><br/>           So 1 and 12 are both factors of 12<br/> <math>2 \times 6 = 12</math><br/>           So 2 and 6 are both factors of 12<br/> <math>3 \times 4 = 12</math><br/>           So 3 and 4 are both factors of 12         </p> |



3 lots of 4 or 4 lots of 3

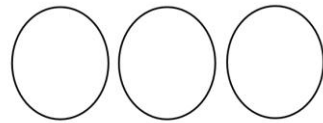


1 lot of 12 or 12 lots of 1

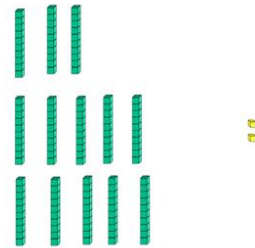


Dividing 2-digit and 3-digit numbers by a single digit, using partitioning

$$132 \div 3 =$$

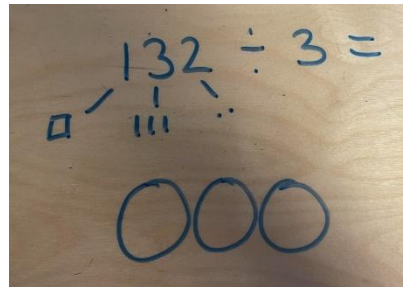


They should quickly notice that 1 hundred cannot be shared between 3 equally as it is so they will need to exchange the 1 hundred for 10 tens.

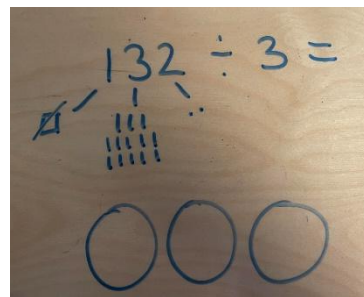


Highlight that we still have our dividend 132

using pictorial representations



The children should create the dividend using dienes.



Recognising that they need to exchange 1 hundred for 10 tens.

$$132 \div 3 =$$

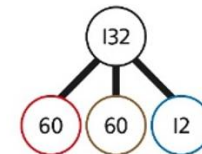
$$100 \div 3 = 30 \text{ with } 10 \text{ left over}$$

$$40 \div 3 = 10 \text{ with } 10 \text{ left over}$$

$$12 \div 3 = 4$$

$$30 + 10 + 4 = 44$$

The children could begin to use non-standard or 'flexible' partitioning, making decisions about appropriate partitioning based on the division required.

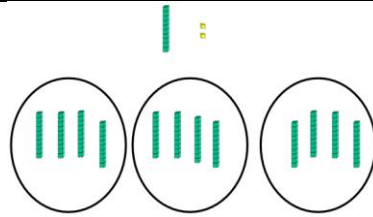


$$60 \div 3 = 20$$

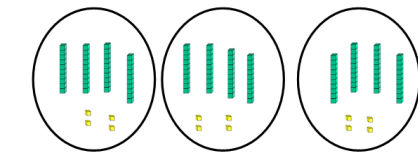
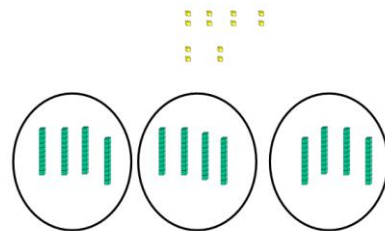
$$60 \div 3 = 20$$

$$12 \div 3 = 4$$

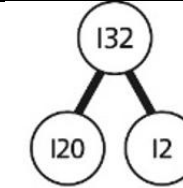
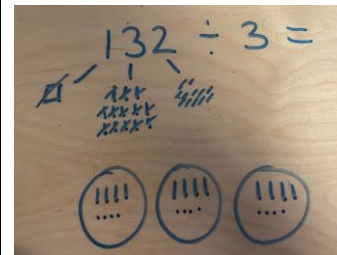
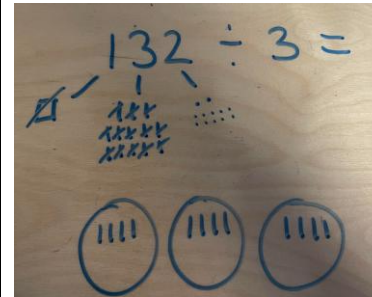
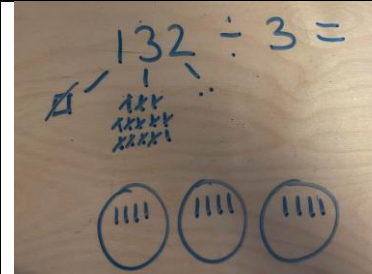
$$20 + 20 + 4 = 44$$



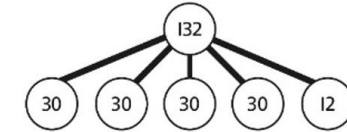
Now we have 4 tens in each group and 1 ten and 2 ones remaining. Another exchange is required, this time they need to exchange 1 ten for 10 ones.



Are the groups equal? How many are there in each group?  
 $132 \div 3 = 44$



$120 \div 3 = 40$   
 $12 \div 3 = 4$   
 $40 + 4 = 44$



$30 \div 3 = 10$   
 $30 \div 3 = 10$   
 $30 \div 3 = 10$   
 $30 \div 3 = 10$   
 $12 \div 3 = 4$   
 $10 + 10 + 10 + 10 + 4 = 44$

The children should begin to understand that different partitions can be used to complete the same division.

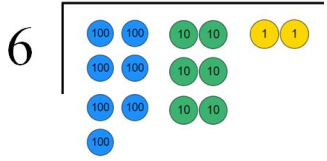
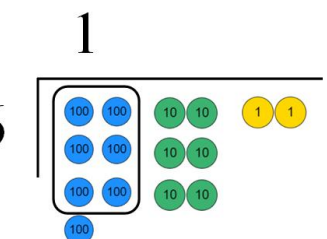
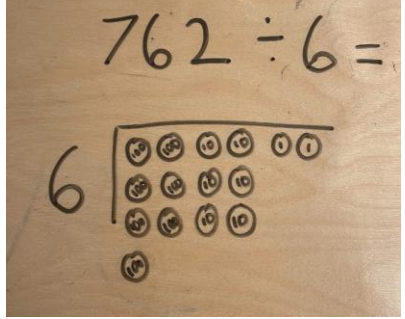
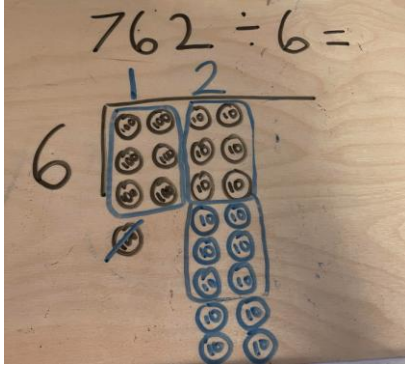

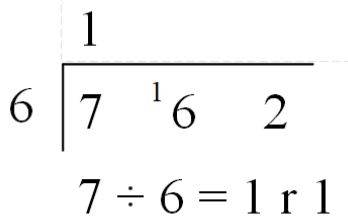
**Vocabulary**

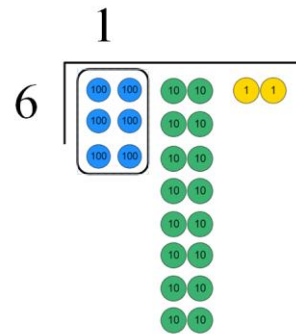
Share, share equally, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, remaining, remainder, divisor, dividend, quotient, product, division facts, inverse, factors,

**Stem Sentences**

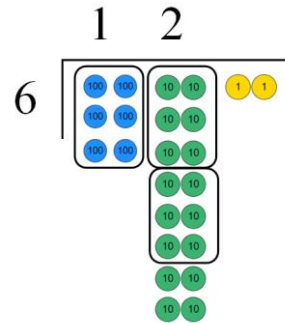
\_\_\_ tens are one ten each. That's \_\_\_\_.  
 \_\_\_ tens are two tens each. That's \_\_\_\_.  
 There are \_\_\_ tens left over.  
 Exchange the remaining tens for ones.  
 \_\_\_ tens and \_\_\_ one is equal to \_\_\_\_ ones.  
 \_\_\_ ones divided between \_\_\_\_  
 is equal to \_\_\_ ones each.  
 \_\_\_ tens and \_\_\_ ones makes \_\_\_\_

Year 5

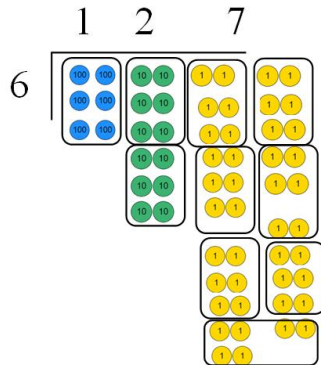
| Objective & Strategy                                       | Concrete  | Pictorial  | Abstract   |
|--|---|--|--|
| <p>Divide numbers up to 4 digits by a one-digit number</p> | <p><math>762 \div 6 =</math></p>  <p>Unlike the other 3 operations, when dividing, we start with the largest digit in value first.</p>  <p>How many lots of or groups of 6 can be made from 700? The children should write the number above the hundreds. They will also recognise that they have 1 hundred remaining. This needs to be exchanged for 10 tens.</p> |  <p>The children should create the dividend using pictorial representations of the number.</p>  |  <p>The children should set the calculation out correctly using a bus stop. Then start with the largest digits in value first (the hundreds)</p>  <p><math>7 \div 6 = 1 \text{ r } 1</math></p> <p>The children might find it helpful to write the calculation below so that they are able to identify what is remaining more easily. They must realise that they are not calculating <math>7 \div 6</math>, they are calculating <math>700 \div 6</math> but as due to where we place the answer, we can calculate it as 7 hundreds.</p> |



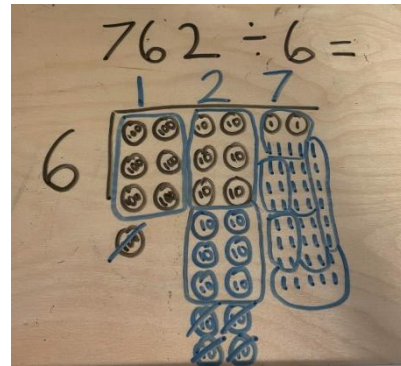
They will then identify that 2 groups of six can be made from 16 tens.



They will then need to exchange 4 tens for 40 ones.



Starting with the hundreds, identifying how many groups of 6 can be made and exchanging the remainder.



$$6 \overline{) 7 \text{ } ^1 6 \text{ } ^4 2}$$

$$7 \div 6 = 1 \text{ r } 1$$

$$16 \div 6 = 2 \text{ r } 4$$

$$6 \overline{) 7 \text{ } ^1 6 \text{ } ^4 2}$$

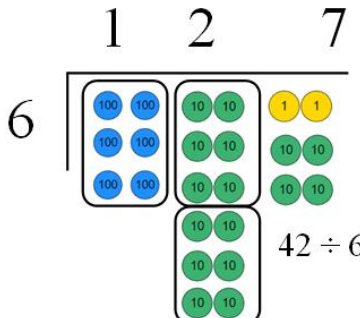
$$7 \div 6 = 1 \text{ r } 1$$

$$16 \div 6 = 2 \text{ r } 4$$

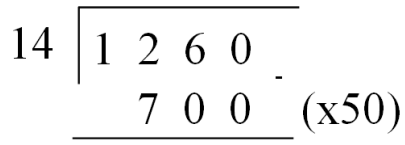
$$42 \div 6 = 7$$

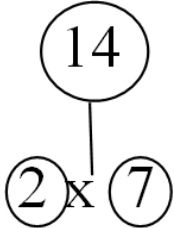
The children could use a more efficient method when confident

$$6 \overline{) 7 \text{ } ^1 6 \text{ } ^4 2}$$

|  |  |  |  |
|--|--|--|--|
|  | <p>If the children are confident with their multiplication tables, they could opt out of the exchange and simply move the 4 tens over, creating 42 and knowing that <math>42 \div 6 = 7</math></p>  |  |  |
|--|--|--|--|

|   |   |
|---|---|
| Vocabulary  | <b>Stem Sentences</b>   |
| <p>Share, share equally, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, remaining, remainder product, division facts, factors, dividend, divisor, quotient, inverse, formal written method.</p> | <p>       ____ <i>tens are one ten each. That's</i> ____.<br/>       ____ <i>tens are two tens each. That's</i> ____.<br/> <i>There are</i> ____ <i>tens left over.</i><br/> <i>Exchange the remaining tens for ones.</i><br/>       ____ <i>tens and</i> ____ <i>one is equal to</i> ____ <i>ones.</i><br/>       ____ <i>ones divided between</i> ____<br/> <i>is equal to</i> ____ <i>ones each.</i><br/>       ____ <i>tens and</i> ____ <i>ones makes</i> ____<br/> <i>Each child gets</i> ____ <i>marbles.</i> </p> |

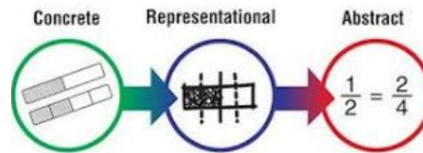
| Year 6   |  |   |   |
|--|--|---|---|
| Objective & Strategy   | Concrete                                   | Pictorial   | Abstract  |
| <p>Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate</p> | <p>Understanding and using known facts</p> | <p><math>1260 \div 14 =</math></p> <p>The children will need to identify the useful facts or 'best friends'</p> <p> <math>1 \times = 14</math><br/> <math>2 \times = 28</math> </p> |  |

|  |   |  |   |
|--|---|--|---|
|  |   | <p>5 x = 70 (using ten times the number to find 5 times the number)</p> <p>10 x = 140</p> <p>20 x = 280</p> <p>50 x = 700</p> <p>100 x = 1400</p>  | <p>Using the known facts, the children should use repeated subtraction to identify how many lots of 14 are in 1260/how many groups of 14 can be made from 1260.</p> $  \begin{array}{r}  14 \overline{) 1260} \\  \underline{700} \quad (\times 50) \\  4 \overline{) 560} \\  \underline{280} \quad (\times 20) \\  280 \quad (\times 20) \\  \underline{000}  \end{array}  $ <p>Calculating the answer by adding the numbers in brackets.</p> |
|  | <p>Understand that division by factors can be used when dividing by a number that is not prime.</p> | <p>1260 ÷ 14 =</p>  <p>Identifying a factor pair of the divisor.</p> <p>Complete the first step, dividing the dividend by one factor</p> <p>1260 ÷ 2 = 630</p> | <p>Identify a factor pair and divide by one of the factors first.</p> $  \begin{array}{r}  0630 \\  2 \overline{) 1260} \\  \underline{1260}  \end{array}  $ <p>Then divide the answer by the second factor.</p>  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |   |
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|---|
|  |  | <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">630</div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">630</div> <p>Then divide this by the second factor.</p> <div style="display: flex; align-items: center; justify-content: center;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td> </td><td> </td><td> </td><td> </td><td> </td><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td><td> </td><td> </td><td> </td><td> </td><td> </td></tr> </table> <span style="margin-left: 10px;">630 ÷ 7</span> </div> |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $  \begin{array}{r}  090 \\  7 \overline{)630} \\  \underline{63} \phantom{0} \\  0  \end{array}  $ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |   |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |   |

|            |                       |
|------------|-----------------------|
| Vocabulary | <b>Stem Sentences</b> |
|------------|-----------------------|


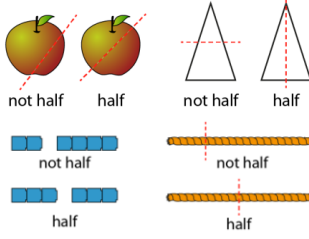


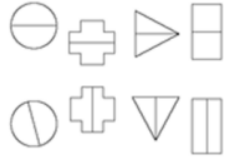
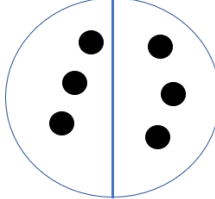
|  |  |
|--|--|
| <p>Share, share equally, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, remaining, remainder product, division facts, factors, dividend, divisor, quotient, inverse, formal written method, factor pairs</p> | <p style="text-align: center;"> <math>\underline{\quad}</math> hundreds divided by <math>\underline{\quad}</math> is equal to <math>\underline{\quad}</math> hundreds with a remainder of <math>\underline{\quad}</math> </p> <p style="text-align: center;"> <i>Exchange the remainder: <math>\underline{\quad}</math> hundreds is equal to <math>\underline{\quad}</math> tens.</i> </p> <p style="text-align: center;"> <math>\underline{\quad}</math> tens divided by <math>\underline{\quad}</math> is equal to <math>\underline{\quad}</math> tens with a remainder of <math>\underline{\quad}</math> </p> <p style="text-align: center;"> <i>Exchange the remainder tens is equal to <math>\underline{\quad}</math> ones</i> </p> |
|--|--|



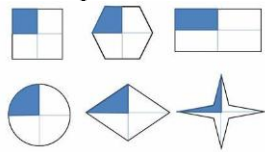
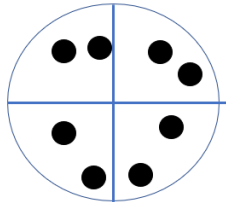
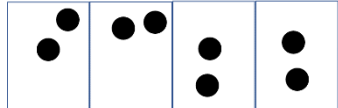


## Fractions

Throughout their time in KS1, children will develop their understanding of whole numbers up to 100, including the numbers zero and one. However, sometimes we can have more than zero but less than one (e.g. half of a cake). This is when we need to use the concept of fractions to tell us about these amounts.

## Year 1

| Objective & Strategy   | Concrete  | Pictorial  | Abstract  |
|--|---|--|---|
| <p>Recognise, find and name a half as one of two equal parts of an object, shape or quantity</p> | <p>To introduce the concept of halving and two halves being <b>equal</b> to each other, model and allow the children time to practise halving physical objects-</p>  <p>This is a good opportunity to explore 'what is half' and 'what is not half'</p>  <p>Children should move on to finding half of small quantities.</p>  <p>Ask the children to find one half of the set of bears but sharing them equally into 2 part or groups-</p>  |  <p>Remind the children of practically sharing concrete resources and explain that they will now move on to using a pictorial representation-</p>  | <p>Beginning to recognise one half as <math>\frac{1}{2}</math><br/>We have one part out of a possible two parts</p> |

|   |   |   |   |
|---|---|---|---|
| <p>Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.</p> | <p>Recap on the process of finding one half. Show the children how to split one whole into 4 <b>equal</b> parts</p>  <p>Children should split their whole into four equal parts-</p>  <p>Rather than drawing four separate circles to share between, this highlights the concept that it is <b>part of a whole</b></p> | <p>Allow the children to see one quarter in a variety of ways pictorially so that they do not associate and 'know' one quarter as one part of a circle-</p>  <p>Give the children a template (or allow them to draw this themselves if possible) and model how to using markings to share the whole into 4 equal parts-</p>  <p>Alternatively -</p>  | <p>Beginning to recognise one quarter as <math>\frac{1}{4}</math><br/>We have one part out of a possible four parts</p> |
| <p>Vocabulary</p>   | <p><b>Stem Sentences</b></p>  |   |   |
| <p>Fraction, equal, part, half, halves, quarter, share, whole</p>                                     | <p>___ is the whole. ___ is a part. ___ is a part.<br/> <i>The .... have/has been divided into 2 equal parts and we have one of those parts</i><br/> <i>The .... have/has been divided into 4 equal parts and we have one of those parts</i><br/> <i>The .... cubes are my whole, so this is half of the whole</i></p>  |   |   |

Year 2

|                      |          |           |          |
|----------------------|----------|-----------|----------|
| Objective & Strategy | Concrete | Pictorial | Abstract |
|----------------------|----------|-----------|----------|

Recognise, find, name and write fractions  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{2}{4}$  and  $\frac{3}{4}$  of a length, shape, set of objects or quantity

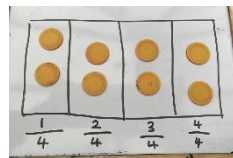
Write simple fractions for example,  $\frac{1}{2}$  of 6 = 3 and recognise the equivalence of  $\frac{2}{4}$  and  $\frac{1}{2}$ .

Recap on the children's learning from Year 1 (above)  
Children should practise sharing cubes or concrete resources into equal groups. For example, to find one third, the children should divide their whole into three equal parts and share the cubes equally between the three parts.



One third of 9 = 3

When finding a non-unit fraction – three quarters, follow the same concept. Ask the children to share their whole into four equal parts. Point – this is one quarter of the (cubes). This makes two quarters of the cubes; how many would we have if we had two quarters? What do we notice about two quarters? We have two parts out of four- make the link that this is HALF! This is three quarters of the cubes. How many do we have if we have three quarters of the cubes?

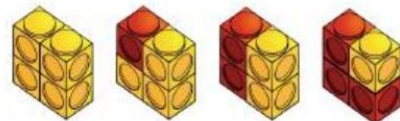


Highlight that we now have **three parts** of the whole, not one

part.

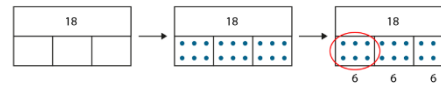
Count in halves up to 10 from any number

Count in quarters up to 10 from any number (Mastery)

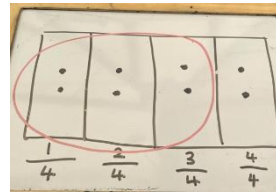


Give the children fraction frames and allow them to place counters into the frame as they count-

Drawing dots/markings to find one third of 18-



When finding non-unit fractions – three quarters.



Work through the same process and then highlight that we now have three parts.

Circle the three parts of the whole- how many in each part? How many do we have altogether? Do we have more or less than one half? How do we know?

$$\begin{aligned} \frac{1}{4} \text{ of } 4 &= 1 \\ \frac{1}{4} \text{ of } 8 &= 2 \\ \frac{1}{4} \text{ of } 12 &= 3 \end{aligned}$$

$$\begin{aligned} \frac{1}{4} \text{ of } 8 &= 2 \\ \frac{2}{4} \text{ of } 8 &= 4 \end{aligned}$$

*What is this equivalent to?*

$$\frac{3}{4} = 6$$

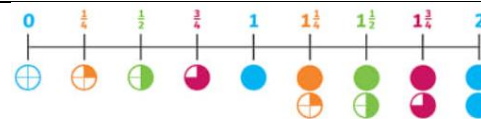
$$\frac{4}{4} = 8 \text{ (one WHOLE)}$$

Children need to begin to make the connection between sharing equally and using division.

If we are finding  $\frac{1}{4}$  of a number, we are dividing that number equally by 4. For example,  $12 \div 4 =$

Children should use the strategies that they have learnt to solve this.





12 is divided into four groups, there are 3 in each group.



Highlight that two quarters is equivalent to one half.

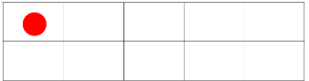
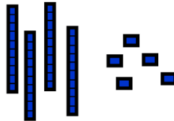

Allow the children time to orally rehearse counting in halves/quarters using a fraction number line to begin with.

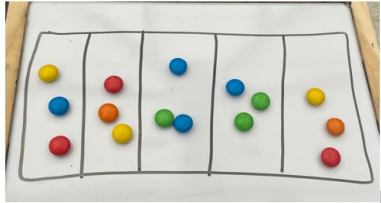
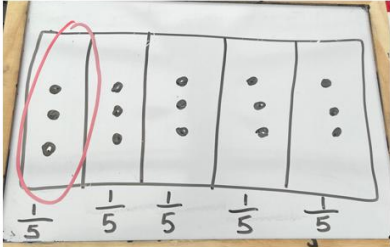
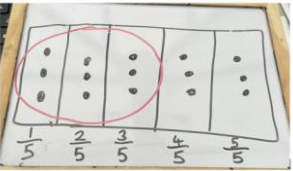

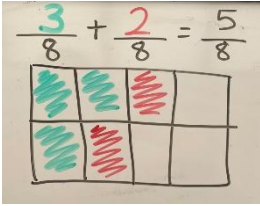

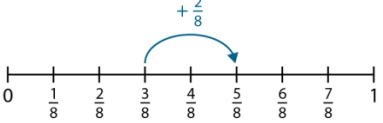
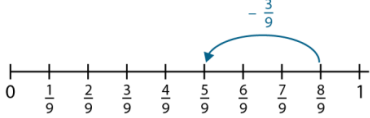
Make the link between the oral count and the pictorial and concrete resources.

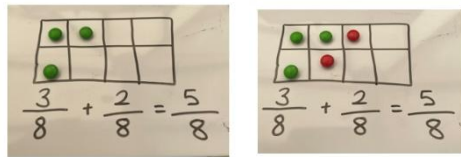
|   |   |  |  |
|---|---|--|--|
|   |  One quarter<br> Two quarters (one half)<br> Three quarters<br> Four quarters (one whole) |  |  |
| Vocabulary  | <b>Stem Sentences</b>   |  |  |
| Divide, equal parts, equivalent, one quarter $\frac{1}{4}$ , one half $\frac{1}{2}$ , one whole, part, share, share equally, one third $\frac{1}{3}$ , group, group equally, three quarters $\frac{3}{4}$ | <p><u>    </u> is divided into <u>    </u> groups. There are <u>    </u> in each group.<br/> If <u>    </u> is the whole then <u>    </u> is a part of the whole.<br/> The whole is <u>    </u>. Half of <u>    </u> is <u>    </u>.</p>  |  |  |

### Year 3

**When teaching fractions to Year 3, be mindful of the times tables the children will already know when moving on to the most efficient method. If the aim is for the children to use a written method to calculate a fraction of a number, consider at least starting with familiar times tables to allow them to draw upon the facts they know.**

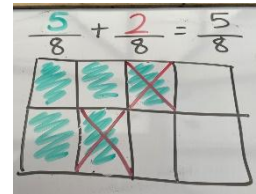
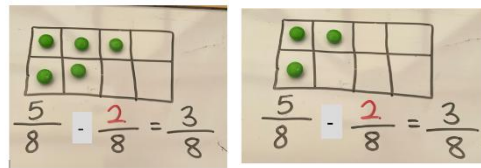
| Objective & Strategy   | Concrete  | Pictorial   | Abstract  |
|--|---|---|---|
| Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 |  <p>Model how to count in tens using a tens frame and counters. Each time a counter is added, count on –<br/> One tenth <math>\frac{1}{10}</math><br/> Two tenths <math>\frac{2}{10}</math></p> <p>Look at the relationship between tenths and 1 whole-</p>  | <p>Children to practise with pictorial representations-</p>  | Children should recognise one tenth as $\frac{1}{10}$ |

|  |  |   |  |
|--|--|---|--|
|  | <p>In this instance, each one diene represents a tenth. 10 tenths is equal to one whole, 1.</p>  |   |  |
| <p>Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators</p>           | <p>Children should use concrete resources to share the whole into the equal groups e.g.<br/> <math>\frac{1}{5}</math> of 15 =</p>  <p>If we have one fifth, we have one part out of 5.<br/>         If we have <math>\frac{3}{5}</math>, we have three parts out of 5.</p> | <p>Children to use the same structure but use markings.</p>  <p>If we have one fifth, we have 3.<br/>         If we have three fifths-</p>  <p><math>3 + 3 + 3 = 9</math></p> | <p>Follow the guidance for Year 2<br/>         Finding non-unit fraction:<br/>         Children should be taught to write the mathematical statement for the process:<br/> <math>\frac{3}{5}</math> of 15 =<br/> <math>15 \div 5 = 3</math><br/>         As we have three fifths we add –<br/> <math>3 + 3 + 3</math><br/>         Or multiply <math>3 \times 3</math></p> |
| <p>Add and subtract fractions with the same denominator within one whole [for example, <math>\frac{5}{7} + \frac{1}{7} = \frac{6}{7}</math>]</p> |  <p>Adding the first fraction e.g. <math>\frac{3}{8}</math> then the second e.g. <math>\frac{2}{8}</math>. How many eights are there altogether? We have <math>\frac{5}{8}</math>, five eights</p>  | <p>Adding fractions</p>    | <p><math>\frac{3}{8} + \frac{2}{8} = \frac{5}{8}</math></p>     |



This is an effective strategy to highlight how many parts are needed to make the whole.

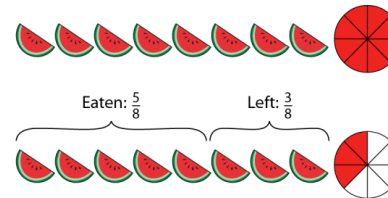
When subtracting, the children should start with the first fraction and then remove the subtrahend.



Subtracting fractions

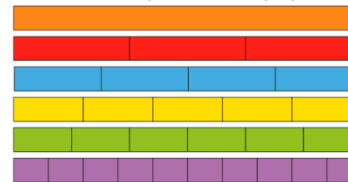


When subtracting from 1 whole e.g.  $1 - \frac{1}{4} =$  show the children the whole and question how many parts we need to remove. What do we need to divide the whole into to subtract one quarter?



Compare and order unit fractions, and fractions with the same denominators

Comparing unit fractions

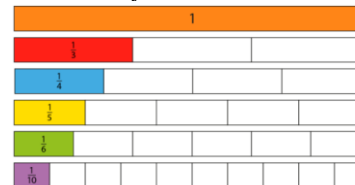


Which coloured strip has the most equal parts? What do you notice about them?

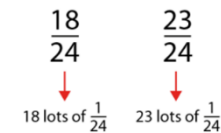
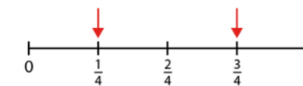
*They are the largest*

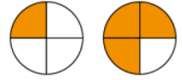

Which coloured strip has the fewest equal parts? What do you notice about them?

*They are the smallest*



comparing non-unit fractions:



|  |  |   |  |
|--|--|---|--|
|  |  | <p>What fraction is each piece of the whole length?</p> <p>Comparing non-unit fractions</p>  $\frac{1}{4} < \frac{3}{4}$  $\frac{3}{8} < \frac{5}{8}$ |  |
|--|--|---|--|

| Vocabulary   | Stem Sentences   |  |     |       |                                 |                    |                       |                    |                                  |                  |
|--|--|--|-----|-------|---------------------------------|--------------------|-----------------------|--------------------|----------------------------------|------------------|
| <p>Divide, equivalent, denominator, numerator, part, unit fraction, non-unit fraction, half <math>\frac{1}{2}</math>, quarter <math>\frac{1}{4}</math>, third <math>\frac{1}{3}</math>, fifth <math>\frac{1}{5}</math>, sixth <math>\frac{1}{6}</math>, seventh <math>\frac{1}{7}</math>, eight <math>\frac{1}{8}</math>, ninth <math>\frac{1}{9}</math>, tenth <math>\frac{1}{10}</math>,</p> | <p><i>If Europe is the whole, then ___ is part of the whole.</i><br/> <i>If ___ is the whole, then ___ is part of the whole.</i><br/> <i>The whole has been divided into ___ equal parts. ___ of the parts has been shaded.</i><br/> <i>(Three one eights) plus (two one eights) equal (five one eights)</i><br/> <i>(6/10) is (six) lots of (1/10), (2/10) is (two) lots of (1/10). I know that (6 + 2 = 8) so, I know that (6/10 + 2/10) equals (8/10)</i><br/> <i>(Three one eights) subtract (two one eights) equal (one one eights)</i><br/> <i>(6/10) is (six) lots of (1/10), (2/10) is (two) lots of (1/10). I know that (6 - 2 = 4) so, I know that (6/10 - 2/10) equals (4/10)</i><br/> <i>When comparing unit-fractions, the greater the denominator, the smaller than fraction</i><br/> <i>When comparing unit-fractions, the smaller the denominator, the greater (or bigger) the fraction</i><br/> <i>We have split our whole into ___ equal parts, so our unit fraction is ___</i><br/> <i>When the numerator and denominator are the same, the fraction is equivalent to one whole.</i><br/> <i>(1/4) is one lot of (1/4). (3/4) is (three) lots of (1/4). I know that (one) is less than (three) so I know (1/4) is less than (3/4)</i></p> | <table border="1"> <thead> <tr> <th style="background-color: yellow;">Say</th> <th style="background-color: lightblue;">Write</th> </tr> </thead> <tbody> <tr> <td>The whole has been divided ....</td> <td>The division bar -</td> </tr> <tr> <td>...into 3 equal parts</td> <td>The denominator: 3</td> </tr> <tr> <td>One of the parts has been shared</td> <td>The numerator: 1</td> </tr> </tbody> </table> | Say | Write | The whole has been divided .... | The division bar - | ...into 3 equal parts | The denominator: 3 | One of the parts has been shared | The numerator: 1 |
| Say  | Write  |  |     |       |                                 |                    |                       |                    |                                  |                  |
| The whole has been divided ....  | The division bar -   |  |     |       |                                 |                    |                       |                    |                                  |                  |
| ...into 3 equal parts  | The denominator: 3   |  |     |       |                                 |                    |                       |                    |                                  |                  |
| One of the parts has been shared   | The numerator: 1   |  |     |       |                                 |                    |                       |                    |                                  |                  |

| Year 4               |          |           |          |
|----------------------|----------|-----------|----------|
| Objective & Strategy | Concrete | Pictorial | Abstract |

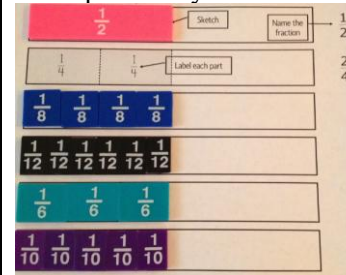
Recognise and show, using diagrams, families of common equivalent fractions

Give the children time to explore equivalent fractions using the Cuisenaire Rods.

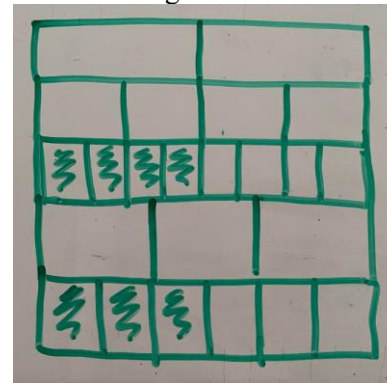


Highlight where an equivalent fraction has been found and where it has not:  
 $\frac{1}{2}$  is equivalent to  $\frac{5}{10}$  but we cannot find a number of fifths that are equivalent to  $\frac{1}{2}$ . Encourage the children to think about why this is the case – because you cannot have half of 5 parts. Can they find any equivalences between the fifths and tenths? E.g.  $\frac{1}{5} = \frac{2}{10}$ .  
 With the second example, the children should be able to recognise that  $\frac{1}{2}$  is equivalent to  $\frac{2}{4}$  and  $\frac{4}{8}$ .  
 This is a good opportunity for the children to explore the relationship here.

Children could build their own fraction walls pictorially:

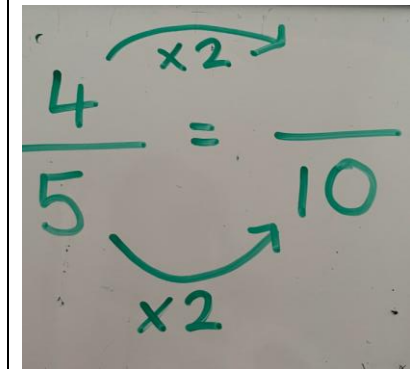


Children can explore drawing their own fraction walls to support with their understanding:

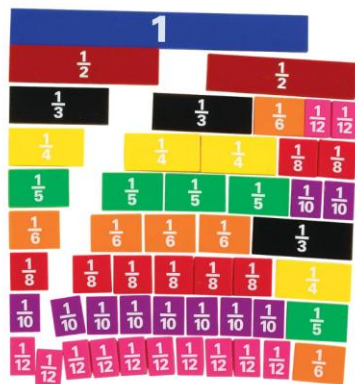


This should highlight the relationship between the numerator and the denominator- what ever we do to the denominator, we must do to the numerator and vice versa.

When the children are ready, they can begin to calculate equivalent fractions using a written method.



Using the 'golden rule'- whatever we do to the numerator, we must do to the denominator and vice versa. In this example, the denominator was five and is now ten. It has been multiplied by 2, therefore, we need to multiply the numerator by 2. So the equivalent fraction will be  $\frac{8}{10}$ .



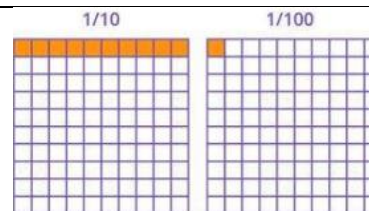
Using fraction tiles and allowing the children to piece this together and find alternative arrangements will support the children's understanding.

Add and subtract fractions with the same denominator

See Year 3 strategies

Recognise and write decimal equivalents of any number of tenths or hundredths (including use of number line)

Use the Numicon shapes and board to introduce the concept. How many ten shapes can be added to the board to fill it? Ask the children to think about the board as one whole (not 100 as they have previously viewed it). The ten shapes are not tenths. So how many tenths make up the whole? Show the children a set number of tenths on the board – there are 5 tenths. Can we write this as a fraction? Say it again five tenths. Now show the children a place value grid – now can we write 5 tenths as a number? 0.5

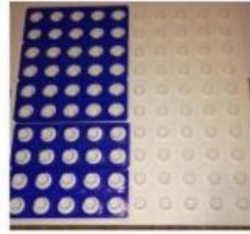


Give the children blank 100 grids and allow them to explore different fraction and decimal equivalencies. Can they colour 4 tenths? What does this look like as a fraction? As a decimal?

The children should then be able to make the link from the pictorial.

$$6/10 = 6 \text{ tenths} = 0.6$$

$$7/100 = 7 \text{ hundredths} = 0.07$$



Repeat with ones – if the ten shape is one tenth. What do we think the one shape will be? It is one hundredth. Model to begin with using 1 one shape and write the equivalent fraction and decimal and then allow the children time to explore with the shapes themselves. If they add a 5 shape and a 2 shape, they have 7 hundredths.  $7/100$  or  $0.07$ . Ensure a place value tracker is available for the children to use.

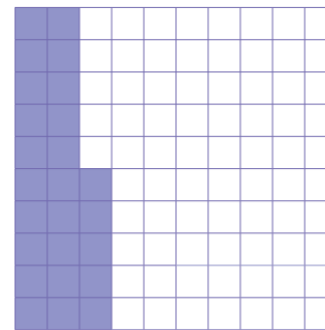
Recognise and write decimal

equivalents to  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$



Again, using the Numicon boards is a good way of demonstrating this concept. Here half of the board is covered. How many individual parts are covered? 50 out of 100. That is a fraction and it is equivalent to  $\frac{1}{2}$ . How many hundredths do we have? 50 hundredths, this is equivalent to 5 tenths. How would we write this as a decimal? Can we cover  $\frac{1}{4}$  of the board? How many part is this? How many hundredths? Can we write this as a decimal? If we know what  $\frac{1}{4}$  is, can we use this to write  $\frac{3}{4}$ ?

The children can continue to explore this using pictorial models by colouring the relevant number of squares to show  $\frac{1}{2}$ ,  $\frac{1}{4}$  and  $\frac{3}{4}$



How many tenths have been shaded (2) and how many hundredths have been shaded (5). Can we write 2 tenths and 5 hundredths using a place value tracker?  $0.25$

The children know and be able to recall the following:

$$\frac{1}{2} = 0.5$$

$$\frac{1}{4} = 0.25$$

$$\frac{3}{4} = 0.75$$

Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths

The children should make the number using coloured counters on a place value grid.

$$302 \div 10 =$$



Explain that when we divide by 10, the place value of the number (and each digit within it) changes too. Each digit will move one place to the right. The children should start with the digits closest to the direction they are moving (the ones). The two ones, will move to the tenth's column. The place value has changed, we now have 2 tenths not 2 ones. We do not have any counters in the ten's column but must remember to use a zero to hold the place of the tens. When moving the three counters in the hundred's column, we change the value from 3 hundreds to 3 tens. Ask the children to write the number they have now:

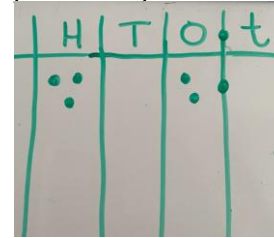


30.2

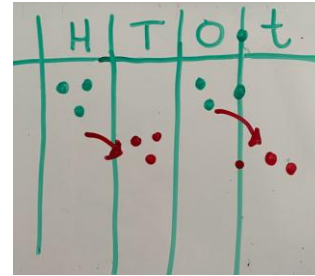
What do they notice about the number they started with and the number they have now?

Place value sliders can also be used but it is important that the children do not become over reliant on these. They need to understand the maths behind this in order to become secure and be able to apply this skill.

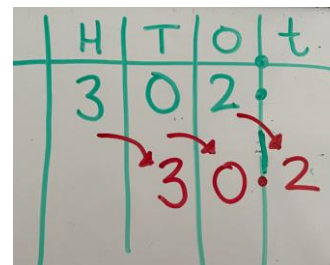
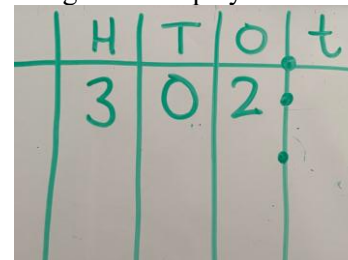
The children should begin with a pictorial representation of the number:



Moving the ones to the tenth's column and the hundreds to the ten's column:



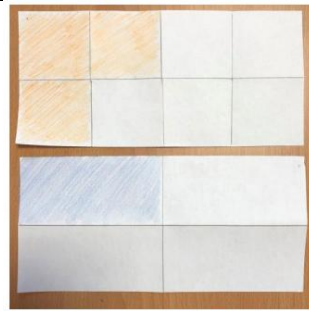
They should then move on to using numerals to represent the number so that they get the opportunity to practise using a zero in play.



When secure, the children should be able to solve the calculations using jottings or completely mentally. Remind the children that we are NOT moving the decimal point and we do NOT add or subtract zeros!

|   |  |  |
|---|--|--|
|   |  |  |
| Vocabulary  | <b>Stem Sentences</b>  |  |
| <p>Fraction, numerator, denominator, whole, equal parts, unit fraction, non-unit fraction, improper fraction, Mixed number fraction, equivalent fractions, simplify, fraction wall, number line, place holder, tenths, hundredths, decimal, decimal place, decimal point,</p> | <p style="text-align: center;"><i>___ tenths and ___ more tenths make ___ tenths.</i></p> <p style="text-align: center;"><i>We had ___ ones. We now have ___ tenths.</i></p> <p style="text-align: center;"><i>___ divided by one hundred is equal to ___</i></p> <p style="text-align: center;"><i>Dividing by one hundred is equivalent to dividing by ten, and then divide by ten again.</i></p> <p style="text-align: center;"><i>When two or more fractions have the same value. We call them equivalent fractions.</i></p> <p style="text-align: center;"><i>The numerator has been scaled up/down by ____ The denominator has been scaled up/down by ____ These fractions are /are not equivalent.</i></p> <p style="text-align: center;"><i>___ is equal to ___ because both the numerator and denominator have been scaled by a factor of ___</i></p> |  |

| Year 5   |   |  |  |
|--|---|--|--|
| Objective & Strategy   | Concrete  | Pictorial  | Abstract   |
| <p>Compare and order fractions whose denominators are all multiples of the same number</p> | <p>It is important for the children to fully understand what a fraction is before they begin comparing. It is common misconception that the larger the denominator the larger the fraction but it is in actual fact the opposite. Show this by dividing the same size rectangle (one below the other) into a different number of equal parts. For example. Divide one into tenths and one into fifths. Shade 1/5 of one and 1/10 of another. Can the children see which is larger? The larger the number of parts, the smaller each part will be.</p> <p>Folding and dividing the same size piece of paper is a good way of exploring this.</p> | <p>Using pictorial representations that may be given to the children to support them to compare the fractions:</p> | <p>It is important for the children to understand some of the key principles behind fractions:</p> <ol style="list-style-type: none"> <li>1. The larger the denominator, the smaller the part</li> <li>2. You can compare unit fractions using this e.g. 1/10 will be smaller than 1/6</li> </ol> <p>This will help to order/compare some fractions.</p> <p>Children should also be aware of the importance of half! If they are able to find fractions that are equivalent to 1/2, they can use this to 'bench mark' against others. If they have 6/10 and 2/12, they</p> |



Using fraction tiles to highlight the relative size of each part can also help



should be able to recognise that  $6/10$  is more than  $1/2$  and  $2/12$  is not.

When comparing  $2/8$  and  $4/12$  the children should find a common denominator. In this case, both are multiples of 4. They can find 24 as a common denominator

$$2/8 = 6/24$$

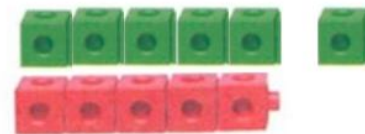
$$4/12 = 8/24$$

Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements  $> 1$  as a mixed number [for example,  $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1 \frac{1}{5}$ ]

Using the example of  $6/5$   
Create the whole using cubes:

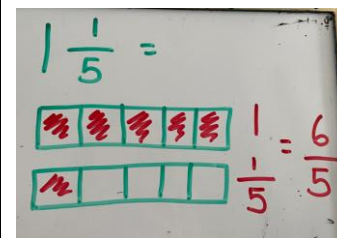


If each cube is 1 fifth and we have 6 fifths, we will need 6 cubes.

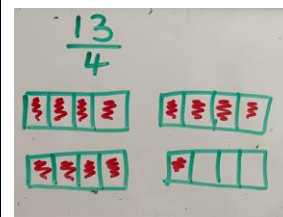


Here we have 1 whole and 1 fifth left so this is  $1 \frac{1}{5}$  or  $6/5$

This can also be completed pictorially by drawing the fractions.



Converting from an improper fraction to a mixed number fraction.



Having practised pictorially, the children should have identified the process. To convert from a mixed number to an improper you need to: Multiply the whole by the denominator Then add the numerator. Work through this process with a pictorial representation to highlight and make the link between the CPA.

$$1 \frac{1}{5} =$$

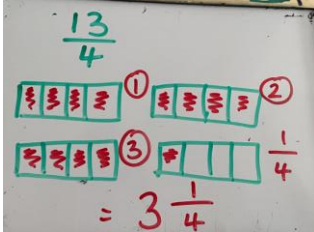
$$1 \times 5 = 5$$

$$5 + 1 \text{ (numerator)} = 6$$

$$\text{We have 6 fifths} = 6/5$$

To convert from an improper fraction to a mixed number, we need to do the opposite. We are dividing and finding out how many wholes we have.

$$13 \div 4 = 3 \text{ with } 1 \text{ remaining}$$




At this stage, the pictorial representations should be drawn by the children and less importance is placed on ensuring they are all identical in size, as long as the children understand they are all equal parts (even if they do not look equal!).


3 wholes and 1 left over  
3 wholes and 1 quarter =  $3 \frac{1}{4}$

Add and subtract fractions with the same denominator and denominators that are multiples of the same number

The children should know that in order to add or subtract fractions the denominators must be the same. The children should be given time to explore how to find common denominators. This can be explored using the Cuisenaire rods




Placing one of the denominators inside and adding the second below. Continue to add the rods representing the denominator until they line up and they have found a common denominator. An example (not in the tracker):



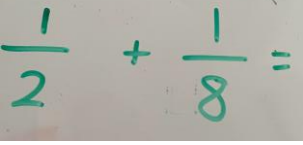
$\frac{1}{2} + \frac{1}{8} =$

Represent the two fractions

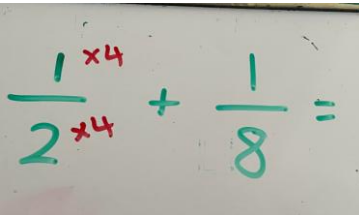


$\frac{1}{2}$        $\frac{1}{2}$        $\frac{1}{8}$

How many eighths do we have altogether? How many eights are there in the half?



First, we need to convert to the same denominator. The children should recognise that 2 can be converted to 8 by multiplying by 4. They then need to remember the golden rule!



Write the new fraction below:

$$\frac{1}{2} + \frac{1}{8} =$$

$$\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$$

Know that we add the numerators but the denominator stays the same.

Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

$$3 \times \frac{3}{4}$$


Explain to the children that multiplication is 'lots of' so we have 3 'lots of'  $\frac{3}{4}$

This can be solved by using repeated addition

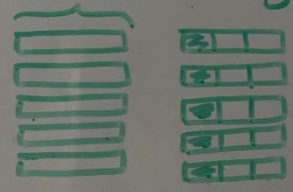
$$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} =$$

How many quarters?

To show this pictorially:

$$3 \times \frac{3}{4} = \frac{9}{4}$$


When multiplying a whole number by a mixed number fraction:

$$5 \times 1 \frac{1}{3} =$$


$5 \times 1 \frac{1}{3} =$   
We have 5 wholes  
and  $\frac{1}{3}$  five times

$$3 \times \frac{3}{4} = \frac{9}{4}$$

In the second example:

$$5 \times 1 \frac{1}{3} =$$

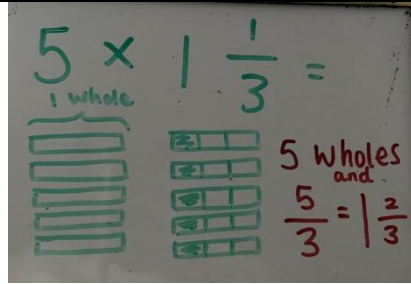
Write 5 wholes as a fraction =  $\frac{5}{1}$

Convert the mixed number fraction into an improper fraction =  $\frac{4}{3}$

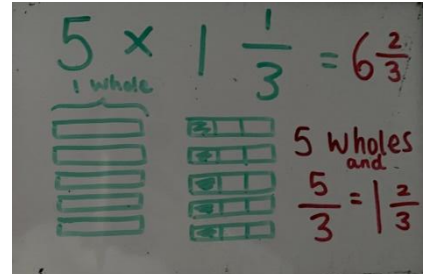
Multiply them together:

$$\frac{5}{1} \times \frac{4}{3} = \frac{20}{3}$$

We can then convert this into a mixed number = 6 wholes and  $\frac{2}{3}$



5 lots of 1/3 = 5/3  
This is equal to 1 whole and 2/3



Read and write decimal numbers as fractions [for example,  $0.71 = \frac{71}{100}$ ]

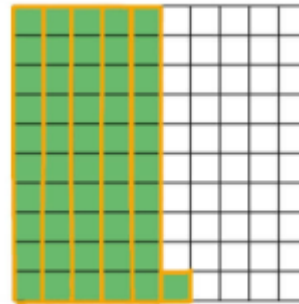
Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal.

Allow the children to explore the concept of percent by using the Numicon shapes and boards. Ask the children to create a pattern on the board using different shapes. Can they write the fraction of blue/red/yellow/green shapes?



So, in this example 18/100 are yellow. This is the fraction. As percent means 'per cent' or 'per hundred' this is equivalent to 18%. In order to write this as a decimal, it needs to have tenths and hundredths. In this case, there is 1 tenth and 8 hundredths = 0.18

This activity could be completed pictorially to embed the concept.



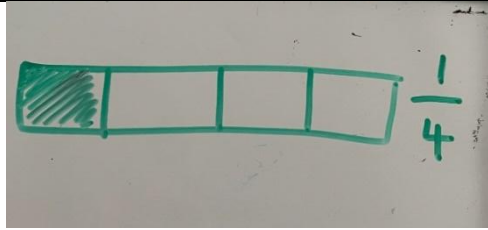
How many parts are coloured? 51/100  
51% = 0.51

Recognise the pattern:  
 $71/100 = 71\% = 0.71$

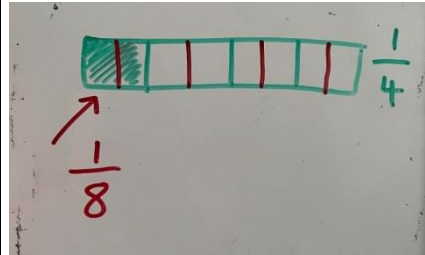
$4/100 = 4\% = 0.04$

|   |  |  |  |
|---|--|--|--|
|   | If the fraction was a 1-digit number e.g. 4/100, highlight that this is 4 hundredths and should be recorded correctly using a place value tracker so it will be 0.04<br>NOT 0.4  |  |  |
| Vocabulary  | <b>Stem Sentences</b>  |  |  |
| Fraction, numerator, denominator, unit fraction, non-unit fraction, improper fraction, mixed number fraction, equivalent, common denominator, decimal, decimal point, decimal place, tenths, hundredths, thousandths, percentage, per cent, product, simplify | <p><i>The whole is divided into ___ equal parts and we have ___ of those parts.</i></p> <p><i>When a whole number is multiplied by a proper fraction, it makes the whole number smaller</i></p> <p><i>The denominator is ___. This means that each whole has been split into ___ equal parts. ___ parts make each whole.</i></p> <p><i>The numerator is ___. This means there are ___ equal parts. It is possible to make ___ full groups of ___ quarters and there are ___ more quarters.</i></p> <p><i>Our unit is ___ so we will be thinking about groups of ___. There are ___ in one whole.</i></p> <p><i>How many groups of in ___ groups and ___ more ___</i></p> |  |  |

| Year 6  |   |  |          |
|---|---|--|----------|
| Objective & Strategy  | Concrete  | Pictorial  | Abstract |
| Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions                               | See above (Year 5)  |  |          |
| Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ ] | The children can see the multiplication as 'of'. So in the example $\frac{1}{2} \times \frac{1}{4}$ the children can see this as finding $\frac{1}{4}$ of $\frac{1}{2}$ | $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$<br>Multiply the numerators = 1<br>Multiply the denominators = 8 |          |

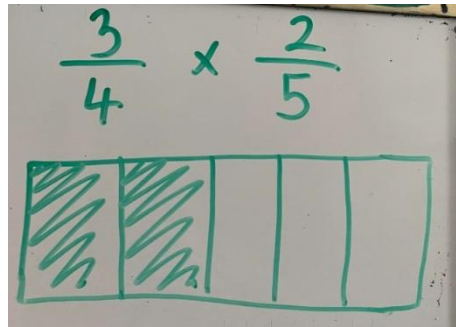


Here is  $\frac{1}{4}$ . If we find  $\frac{1}{2}$  of this, we will need to halve each quarter.



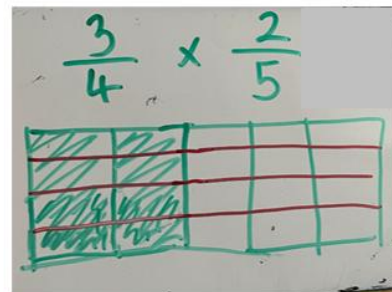
Now we have 1 part out of a possible 8.

The same applies when multiplying non-unit fractions:

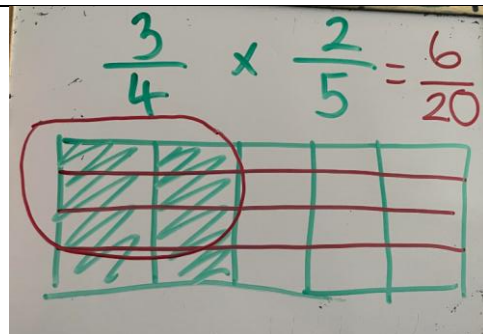


We are finding  $\frac{3}{4}$  of  $\frac{2}{5}$   
First, show the two fifths

Then divide the fifths into quarters:

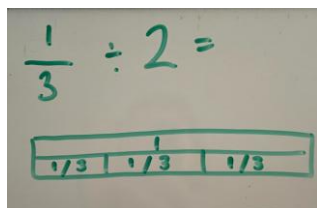


We have 3 of the quarters:

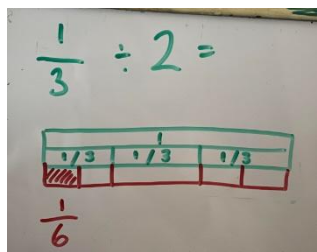


Divide proper fractions by whole numbers [for example,  $\frac{1}{3} \div 2 = \frac{1}{6}$ ]

In the example  $\frac{1}{3} \div 2 =$   
First, we need to understand  $\frac{1}{3}$  in relation to the whole:



Then we need to divide the thirds by 2.



Here we have 1 part out of a possible 6 as we need to include all of the parts from the whole.

$\frac{1}{3} \div 2 =$   
Write them both as fractions:  
 $\frac{1}{3} \div \frac{2}{1} =$   
Keep the first fraction:  
 $\frac{1}{3}$   
Flip the second:  
 $\frac{1}{2}$   
Change division to multiplication  
 $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

This procedure should only be introduced once the children are confident with the process involved when fractions are divided by whole numbers by using pictorial representations.

**Vocabulary**

**Stem Sentences**

Fraction, numerator, denominator, unit fraction, non-unit fraction, improper fraction, mixed number fraction, equivalent, common denominator, decimal, decimal point, decimal place, tenths, hundredths, thousandths, percentage, per cent, product, simplify

*If we divide into \_\_\_ equal groups, then each of the groups is because \_\_\_  $\div$  \_\_\_ = \_\_\_  
A fraction can be converted into a decimal by dividing the numerator by the denominator.  
To find 10% of a number, divide the number by 10  
To find 1% of a number, divide the number by 100*

*In order to convert a percentages to a fraction, first convert it to a fraction with a denominator of 100 then simplify.*

